

# A Model for Managing Interest Rate Risk in the Banking Book: A comparison between a modified earnings-based approach and economic value approach

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**Abstract.** The management of interest rate risk in the banking book is a significant issue for banks which earn profits by making loans and investing in securities using deposit funding. We have many difficulties to evaluate some components in the banking book, such as cancelling its position, estimating cash flow and maturity of core deposit, and interest rates applied incompletely linking to market interest rate. In practice, interest rate risk in the banking book has been discussed from the perspectives of both economic value and earnings. However, there is no clear consensus of opinion about which perspective is more important for banks. In order to manage interest rate risk in the banking book, we develop a mathematical model that takes into account the characteristics of the complex banking book with respect to pass-through rates and core deposits. The model also considers dependency relationships between interest rate risk and credit risk. We examine the model through various sensitivity analyses using the Monte Carlo simulation, and we clarify the relationship between an economic value approach and earnings-based approach involving expected market value variations. To the best of our knowledge, there are no detailed studies conducted using such a practical model to manage interest rate risk in the banking book. From the results of the analyses, we find that, to properly reflect the characteristics of the banking book in its risk management, we need to manage the risk from the earnings-based approach as well as the economic value approach. In addition, we find that the pass-through rates between market interest rates and deposit or loan interest rates are sources of the profitability of banks and the cash flow of core deposits expected using the model make a great impact on interest rate risk in the banking book. In particular, we show that banks may be affected by a type of estimation model of core deposits when market interest rates change.

**Keywords:** interest rate risk, banking book, modified earnings-based approach, economic value approach, core deposits

## 1. INTRODUCTION

The management of interest rate risk in the banking book is a significant issue for banks which earn profits by making loans and investing in securities using deposit funding. Basel Committee on Banking Supervision (BCBS) has discussed how to manage this risk as one of the significant issues on international bank supervision for a long time, and attempts to revise the methodology in the

regulatory framework. We have many difficulties to evaluate some components in the banking book, such as cancelling its position, estimating cash flow and maturity of core deposit, and interest rates applied incompletely linking to market interest rate. In practice, interest rate risk in the banking book has been discussed from the perspectives of both economic value and earnings. However, there is no clear consensus of opinion about which perspective is more important for banks (Yoshifuji 1997).

In general, we define the economic value approach and earnings-based approach as follows. Furthermore, two kinds of earnings-based approaches are defined according to the difference in recognition of income.

- **Economic value approach**
  - We evaluate economic value (present value) for all future cash flow generated by current assets and liabilities.
- **Earnings-based approach (only net income)**
  - We evaluate net interest income from the bank accounts (ex. loan and deposit) including new transactions for several years.
- **Earnings-based approach (net income + expected market value variations)**
  - We recognize not only net income but economic value as earnings for several years. Therefore, we evaluate them for all future cash flow generated by both assets and liabilities that banks hold at each period.

Kiyama et al. (1996) introduce an economic value approach into interest rate management in the banking book based on only deposits and loans. We recognize cash flow from a long-term perspective in the economic value approach, whereas we recognize it from a short-term perspective in the earnings-based approach. Meanwhile, we focus on the risk associated with the current position from a short-term perspective in the economic value approach, whereas we focus on the risk associated with the balance sheet variations in the future from a long-term perspective in the earnings-based approach. In the banking book, it is important to evaluate the earnings, especially from the viewpoint of going concern assumption. Yoshifuji (1997) compares the interest rate risk management of bond portfolio in the banking book using the Earning at Risk (EaR) model with the extended Value at Risk (VaR) model in the earnings-based approach.<sup>1</sup> It is shown that a short-term simulation using only realized earnings misleads the bank. We can classify these three approaches from the perspective of cash flow and risk recognition as Table 1.

Memmel (2014) replicates the banking book using a bond investment strategy, and evaluate interest rate risk with respect to the net interest income (only realized earnings) and the present value. The result shows two factors are highly correlated, however, the changes in the term structure of interest rate give quite different impacts on them. Therefore, we cannot evaluate risk appropriately by the stress test according to only the present value.

<sup>1</sup> Only realized earnings are included in the EaR model, whereas the market value change is also included in the extended VaR model.

Table 1: The characteristic of each approach

		Risk recognition	
		Short-term	Long-term
Cash flow	Short-term		Earning-based approach (only net income)
	Long-term	Economic value approach	Earning-based approach (net income + expected market value variations)

In addition, as the market becomes more complex, banks need to introduce an "integrated risk management" which is more sophisticated, in order to improve profitability as well as soundness. Mikuni and Hibiki (2014) propose a multi-period optimization model for the integrated risk management, which involves dependency relationships between interest rate risk and credit risk with a copula.

We conduct sensitivity analyses to examine the model, using the Monte Carlo simulation, and we clarify the relationship between an economic value approach and earnings-based approach involving expected market value variations, as well as Yoshifuji (1997). In this paper, we call the earnings-based approach "**modified earnings-based approach**". To the best of our knowledge, there are no detailed studies conducted using such a practical model to manage interest rate risk in the banking book. According to the analyses, we find that the pass-through rates between market interest rates and deposit or loan interest rates are sources of the profitability of banks, and the cash flows of core deposits are different among the estimation models, and they make a great impact on interest rate risk in the banking book. The main characteristics and contributions of our study are as follows.

- We clarify the relationship between an economic value approach and modified earnings-based approach. We find that we can evaluate the risk on the banking book appropriately in the modified earnings-based approach, and the result in the economic value approach misleads the bank management due to lack of the future transactions perspective.
- We develop a mathematical model involving dependency relationships between interest rate risk and credit risk using a copula, and analyze several cases comprehensively.
- We conduct sensitivity analyses for the various term-structure of interest rate using the Nelson-Siegel model.
- We clarify the characteristics of the banking book with respect to pass-through rates and core deposits. We find that pass-through rates are sources of the profitability of banks.
- We estimate expected cash flows of core deposits, using the Kamitake-Hibiki model (2011). We compare the results between the model and the Standardized

Approach in the regulation, and we find that it is important to select the model to estimate the cash flow of core deposits because it makes a great impact on interest rate risk in the banking book.

We show Table 2 to compare our paper with the previous studies and clarify the differences.

This paper is organized as follows. In Section 2, we briefly explain the overview of model structures for managing the interest rate risk in the banking book. In Section 3, we show the numerical analyses for a hypothetical commercial bank. We conduct the sensitivity analysis of the term structures of interest rates, pass-through rates and rollover rates. We also evaluate the model risk. Section 4 provides our concluding remarks.

Table 2: Comparison of our paper with the previous studies

	Kiyama et al.(1998)	Yoshifuji (1997)	Memmel (2014)	Mikuni & Hibiki (2014)	Our Paper
Balance Sheet					
Loan & Deposit	○	×	×	○	○
Bond	×	○	○	○	○
Economic Values	○	×	○	○	○
Earnings (*1)	×	○	△	△	○
Core Deposit	×	×	×	○	○
Prepayment(*2)	○	×	×	×	△
Term structure	○	○	○	×	○
Dependency (*3)	×	×	×	○	○

(\*1) △: Only net income, ○: net income + expected market value variations

(\*2) △: PSJ model which is not dependent on interest rate is utilized

(\*3) Dependency relationship between interest rate risk and credit risk

## 2. MODEL STRUCTURE FOR BANKING BOOK

### 2.1 Model Structure

We construct risk factor models for generating macroeconomic scenarios and cash flow models for asset and liabilities. The model structure of our model is shown in Figure 1.

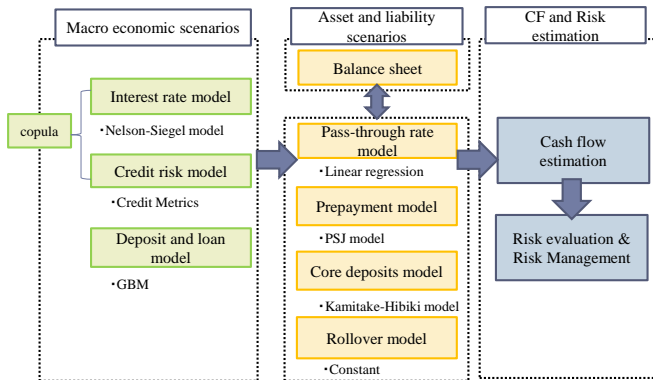


Figure 1: Model structure to manage the banking book

We estimate each parameter by using the monthly data from Oct 2004 to Sep 2014, and the future cash flow utilizing these models. We evaluate the downside risk of the economic value and that of the modified earnings.

### 2.2 Risk Factor Models for Macroeconomic Scenarios

We utilize the dynamic Nelson-Siegel model, which is modified by Diebold and Li (2006), to express term structures of interest rate. The model includes three factors that express components of a yield curve called "level", "slope" and "curvature". The spot rate of maturity  $\tau$  at time  $t$  is expressed as,

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t} \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_{3,t} \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \quad (1)$$

where  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  denote the values of "level", "slope" and "curvature", respectively. We assume that these factors are described by the AR(1) model to express variations of term structure in the future as follows.

$$\beta_{k,t} = c_k + \varphi \beta_{k,t-1} + \varepsilon_{k,t} \quad (k = 1, 2, 3) \quad (2)$$

The model parameters are estimated by using LIBOR and swap rates.

We employ *Credit Metrics* provided by J.P. Morgan to express transitions of credit ratings of borrowers group. The model is referred to as a structure model. We utilize the average transition probability in Jun 2014 published by Rating & Investment Information, Inc.

We assume that there is a copula dependency between three factors of interest rate and credit ratings. We examine three kinds of copulas; Gaussian, Student's t, and mixed-Gaussian copula.

In addition, we assume that both amounts of deposits and loans follow the Geometric Brownian motion (GBM) and the fluctuation rates are correlated.

### 2.3 Cash Flow Models for Asset and Liabilities

We explain the scenario generation models for assets and liabilities in the banking book. We utilize a core deposit model which is called Kamitake-Hibiki model (2011). The model describes the fixed deposit to liquid deposit ratio  $\rho_t := TD_t / LD_t$  to express money transfer between time deposits and liquid deposits, where  $TD_t$  denotes a time deposit and  $LD_t$  denotes a liquid deposit. Therefore, we can simultaneously estimate both amounts of time deposits and liquid deposits. The total amounts of deposits are also calculated using the Geometric Brownian motion (GBM). The ratio  $\rho_t$  at time  $t$  is expressed as,

$$\rho_t = (\alpha_1 \ln r_t - \alpha_2) t + \alpha_3 \ln r_t + \alpha_4 \quad (3)$$

where  $r_t$  is a short-term interest rate.

We utilize the PSJ (Prepayment Standard Japan) model which is a standard model to estimate prepayment rates of mortgages by the Japan Securities Dealers Association. Annual Conditional Prepayment Ratio,  $CPR_m$ , is a simple deterministic function with respect to  $m$  months, and defined as follows:

$$CPR_m(\%) = \min\left(\frac{6}{60} \times m, 6\right) \quad (4)$$

Lastly, we assume that rollover rate is constant, because it is difficult to obtain the rollover data and there are no standard models.

## 2.4 Evaluation Measure

We define a modified CVaR ratio as follows. We can utilize the measure for various probability distributions.

We denote the return in period  $T$  as  $R_T^{Net}$ , and the return of risk-free rate as  $R_T^f$ .

$$\text{Modified CVaR ratio} := \frac{E[R_T^{Net}] - R_T^f}{\text{CVaR}[R_T^{Net}] + E[R_T^{Net}]} \quad (5)$$

A confidence level of CVaR is set to be 99%, and the number of simulation paths is 10000 in the numerical analysis.

## 3. NUMERICAL ANALYSES

### 3.1 Simulation Setting

We define the methods specifically for economic value approach and modified earnings-based approach as follows.

- **Economic value approach**
  - We evaluate economic value variations of the current balance sheet occurred by interest rate change within ten days<sup>2</sup>.
- **Modified earnings-based approach**
  - We assume that the balance sheet changes over time, and evaluate earnings plus expected market value variations for the balance sheet updated semi-annually within three years (six periods).

We assume a hypothetical Japanese commercial bank as in Table 3. It has government bonds, corporate bonds, loans and cash in assets, and then liquidity (non-maturity) deposits and time deposits in liabilities.

### 3.2 Outline of Numerical Analyses

<sup>2</sup> The ten days variation is calculated to one third of a month variation, because the value is estimated monthly.

Table 3: Balance sheet (unit of amount; trillion yen)

Asset		Liabilities	
Govt. Bond 10Y	26.4	TD6M Corp.	15.0
Govt. Bond 5Y	105.4	TD6M Individual	58.1
Corp. Bond 3Y H	15.6	TD1Y Corp.	35.0
Corp. Bond 3Y L	15.6	TD1Y Individual	135.6
Loan6M H (Fixed)	61.6	LD Corp.	90.8
Loan6M L (Fixed)	61.6	LD Individual	217.0
Loan5Y H (Fixed)	20.2	Other liabilities	83.6
Loan5Y L (Fixed)	20.2		
Loan5Y H (Floated)	96.0		
Loan5Y L (Floated)	96.0	Equity	
Mortgage 10Y	85.1		
Cash	74.0	Equity	42.8
Total	677.8	Total	677.8

Note: H...High rank, L...Low rank

We conduct the numerical analyses for the following four points.

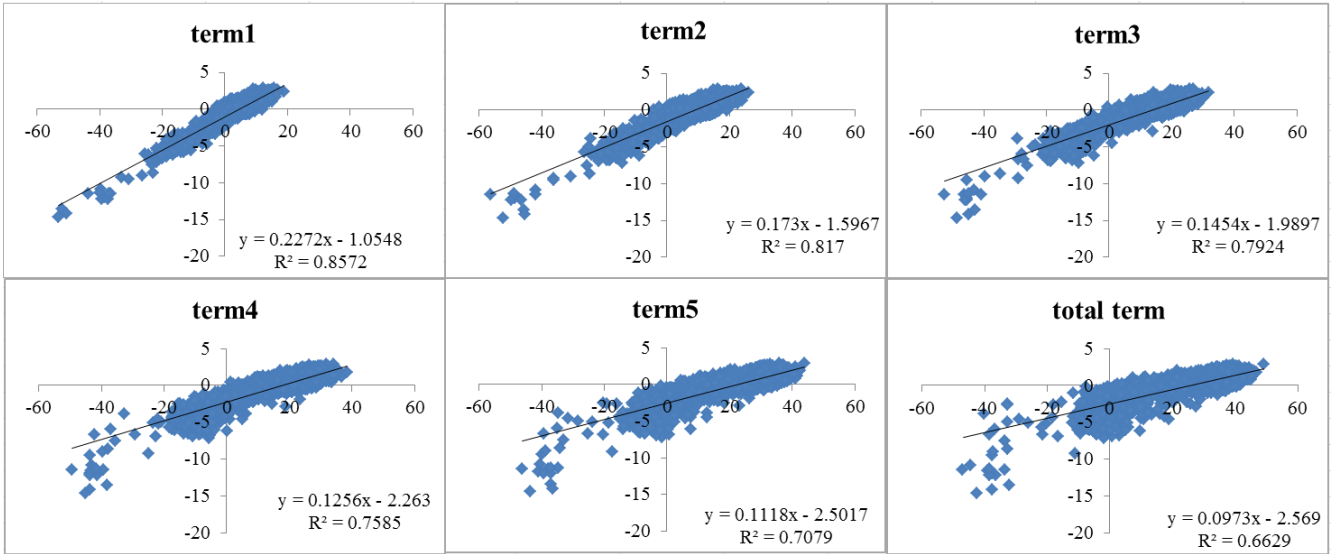
1. A base case
  - We analyze the relationship between an economic value approach and modified earnings-based approach, using the base parameters.
2. Sensitivity of the term structures of interest rates
  - We analyze the sensitivity of the term structure of interest rates to the banking book. We consider four kinds of term structure changes; a parallel shift, shock, steepening and flattening.
3. Sensitivity of the pass-through rates and rollover rates
  - We analyze the sensitivity of the pass-through rates and the rollover rates, which makes a great impact on economic values and modified earnings.
4. Quantitative evaluation of model risk
  - We evaluate the model risk quantitatively. We compare the results among different core deposits models, interest rate models and copulas, respectively.

### 3.3 Base Case

We discuss the relationship between the economic value approach and modified earnings-based approach. We generate the simulation scenarios using the base parameters estimated from the historical data as explained Section 2.2. The estimated parameters of AR(1) model are shown in Table 4.

Table 4: Parameters of AR(1) model

	$c$	$\phi$	$\sigma_\epsilon$	$R^2$
$\beta_1$	0.069	0.96	0.121	0.911
$\beta_2$	-0.062	0.946	0.167	0.893
$\beta_3$	-0.238	0.918	0.329	0.871



(horizontal axis : cumulative modified earnings, vertical axis: economic value variations, units: a trillion yen)  
 Figure 2: Base case: the relationship between cumulative modified earnings and economic value variations

The average transition probability matrix is shown in Table 5.

Table 5: Average transition probability matrix

	AAA	AA	A	BBB	BB	B	CCC
AAA	91.0%	9.0%					
AA	0.8%	93.9%	5.2%	0.1%			
A		1.8%	94.3%	3.7%	0.1%		0.1%
BBB			3.8%	93.4%	2.7%		0.1%
BB			0.3%	7.9%	86.6%	2.6%	
B				0.8%	9.9%	77.0%	12.3%
CCC						4.5%	95.5%

We utilize Gaussian copula in dependency relationships among interest rate risk and credit risk. The parameters of Gaussian copula are shown in Table 6.

Table 6: Parameters of Gaussian copula

	$\beta_1$	$\beta_2$	$\beta_3$	Stock (large)	Stock (small)	
Level	$\beta_1$	1				
Slope	$\beta_2$	-0.86	1			
Curvature	$\beta_3$	-0.24	-0.07	1		
Stock (large)		0.43	-0.45	-0.03	1	
Stock (small)		0.3	-0.34	-0.02	0.91	1

We show the pass-through rates estimated by the linear regression model in Table 7. The determination coefficients are enough high, and we can confirm that the pass-through rate of loan rate is higher than the pass-through rates of deposit rates.

We show that scatter plots of the values in all economic scenarios in Figure 2, where the horizontal axis

Table 7: Pass-through rates

Interest rate	Reference term	Intercept	Pass-through rate	R <sup>2</sup>
Short-term prime rate	6M	1.365	0.716	0.855
Time deposit 6M	6M	-0.012	0.445	0.924
Time deposit 1Y	1Y	-0.012	0.589	0.895
Liquidity deposit	6M	-0.017	0.306	0.891

shows cumulative modified earnings, and the vertical axis shows economic value variations. The same economic values are shown in all graphs whereas the different modified earnings are shown in each cumulative period, respectively. We find the following facts in Figure 2.

- The modified earnings in period 1 are extremely correlated with economic value variations. Therefore, we obtain the similar result in the both approaches. (The perspective of a present B/S)
- The correlations (determination coefficients) between two approaches decline as the earnings are accumulated. Therefore, we obtain the different results in the both approaches. (The perspective of a future B/S updated according to transaction)

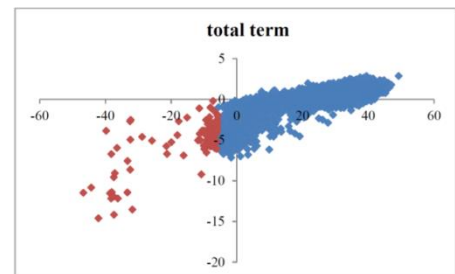


Figure 3: 1% worst-case scenarios in modified earnings where the points are red-colored

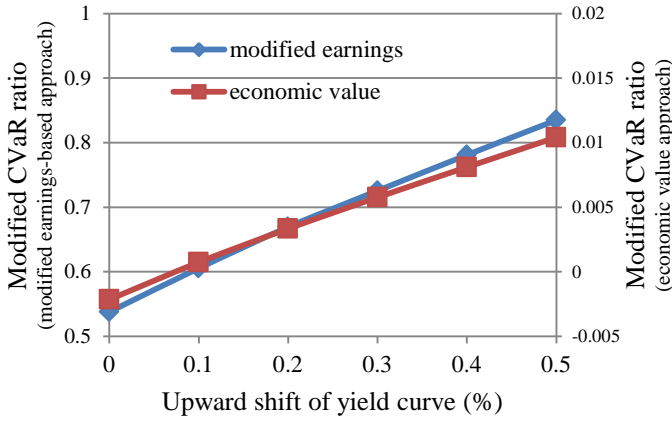


Figure 4: Modified CVaR ratio for different upward shifts of yield curve

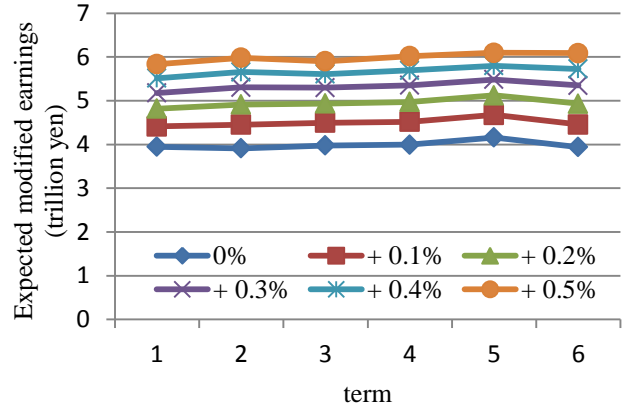


Figure 5: Expected modified earnings for different upward shifts of yield curve

We can manage the income risk comprehensively in the modified earnings-based approach due to the future perspective, compared with the economic value approach. We show the scatter plot of 'total term' again in Figure 3 where the lowest hundred samples of the modified earnings are red-colored. We confirm that the economic value variations are excessively scattering to the 1% worst-case scenarios when the modified earnings extremely fall. The economic values in some scenarios do not fall even when the modified earnings fall. Therefore, the result in the economic value approach misleads the bank due to lack of the future transactions perspective.

Due to space limitations, the histograms are omitted, but we find the distributions of economic values and modified earnings are negatively skewed and have fat tails due to default risk. Therefore, banks have the possibilities of suffering from enormous losses in the worst case scenarios.

### 3.4 Sensitivity of the Term Structures of Interest Rates

Due to space limitations, we describe the results of the parallel shift case. We assume that a parallel shift of yield curve occurs at the constant rate semi-annually for three years. We show the modified CVaR ratio and a fluctuation of the expected modified earnings on each period in Figures 4 and 5, when annual increases in interest rate change from 0% to 0.5%.

Figure 5 shows that the expected modified earnings increase as the interest rate rises. The pass-through rate of loan rate is higher than that of deposit rate, and the interest rate spread between them becomes wider. As a result, the income profit becomes larger than the funding cost when banks roll over the accounts or obtain new contracts. We also find the modified CVaR ratios which

measure the risk-return efficiency also increase in the both approaches. The economic value of asset usually decreases as the interest rate rises, because banks invest many amounts of long-term government bonds. However, the decrease in the liability values remarkably affects the economic value, beyond the decrease in the asset values. Therefore the interest rate changes give a reverse impact on the economic values against the usual situation. We show the maturities of liquid deposit estimated by the Kamitake-Hibiki model for two cases in Table 8<sup>3</sup>.

Table 8: Maturity of liquid deposit (unit: year)

	Base case		Parallel shift 0.5%	
	Corp.	Individual	Corp.	Individual
Expected V.	8.02	8.04	7.53	7.70
99% VaR	5.92	6.23	5.67	6.07

In the base case, the expected maturity is around eight years, and the 99% VaR (Volume at Risk) of maturity is around six years. These are longer than the standard value by BIS (Bank for International Settlements). When the interest rate rises, money is supposed to be transferred from liquid deposits to time deposits. Therefore the maturity is shorter than the base case, but it is still long, compared with the maturity of the time deposits. In addition, the pass-through rate of liquid deposits is low, and it is practically nearly stable. For these reasons, banks can strongly offset the interest rate risk with their liabilities, and the economic value increases as the interest rate rises.

The results indicate that it is important to consider the pass-through rate and maturity of liquid deposits appropriately.

<sup>3</sup> Maturity of liquid deposit  $M_{LD}$  is calculated by using amounts of withdrawal  $X_t$  at time  $t$ , as  $M_{LD} = \sum X_t / LD_0$

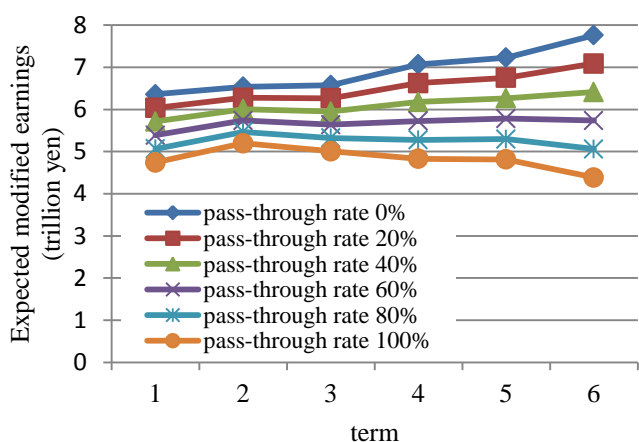


Figure 6: Effect of pass-through rate (deposit rate)

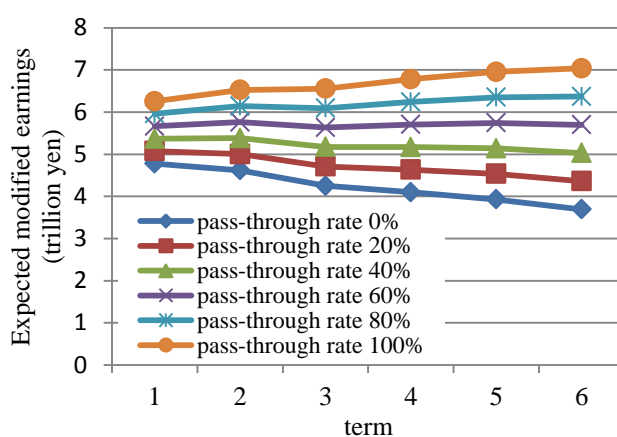


Figure 7: Effect of pass-through rate (loan rate)

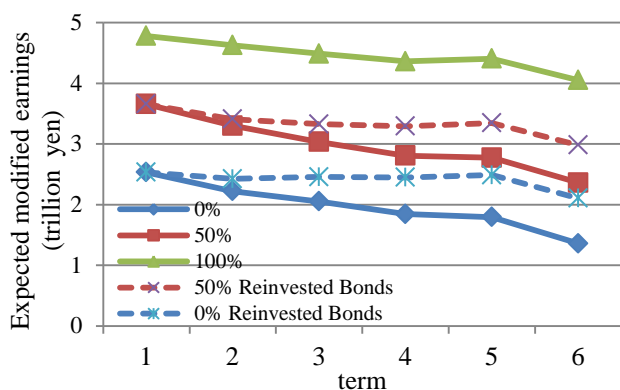


Figure 8: The relationship between the rollover rate of assets and expected modified earnings

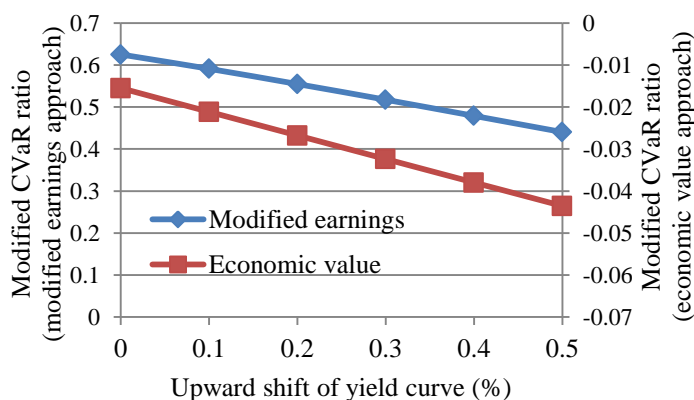


Figure 9: The Standardized approach

Without going into detail, we derive different results in the two approaches when short term rate rises and yield curve is inverted. Whereas the economic value decreases due to the remarkable increase in the asset value with relatively short maturity to the liabilities, the modified earnings increase because the spreads of new transaction are wider. The results indicate that banks with complicated balance sheet need to focus on change of term structure of yield curve.

### 3.5 Sensitivity of the Pass-through Rates and Rollover Rates

We analyze the sensitivities of the pass-through rate and the rollover rate to economic value and earnings.

#### 3.5.1 Sensitivity of Pass-through Rates

The pass-through rates greatly affect economic value and earnings through a spread between deposit and

loan rates, and it is sensitive to the values of the banking book. We conduct the sensitivity analyses to the pass-through rates of time and liquid deposits, and those of loans. Suppose the yield curve is adjusted so that the initial interest rate can be equal in each case. We analyze the case where the interest rate increases annually by 0.5%.

We show the expected modified earnings for different pass-through rates of deposits in Figure 6 and those of loans in Figure 7. As the deposit pass-through rate is low or the loan pass-through rate is high, and the interest rate rises, the interest rate spread expand and banks increase their profit.

#### 3.5.2 Sensitivity of Rollover Rates

Bank profit is affected by the rollover and future contracts. We conduct the sensitivity analysis of loan rollover rate. When the amounts of new loan contracts are constant and we set the rollover rate to be 0%, 50%, and 100% (the base case), we show the result in Figure 8. We

assume that we invest in five-year bond or call loan as the alternative to loans that are not rolled over. When the rollover rate is low, the modified earnings greatly changes. A decline of rollover rate reduces profits, because the profitability of loan is higher than that of bond or call loan in our setting. A fluctuation of rollover rate greatly affects the result through changes of bank assets structure in the modified earnings-based approach.

### 3.6 Quantitative Evaluation of Model Risk

Due to space limitations, we focus on the analysis of core deposit models. In Section 3.3, the interest rate changes of liquid deposit give a great impact on risk management in the banking book. We compare the results among different core deposit models; the Kamitake-Hibiki model vs. the regulatory standardized approach. The regulatory model offers extremely conservative (short) maturity, and sets the maximum to 1.25 years. Suppose the amounts of deposits are equal to those derived by the Kamitake-Hibiki model. We show that the modified CVaR ratios derived by the regulatory approach in Figure 9. The risk-return efficiency reduces as the yield curve shifts largely, and therefore it leads to the opposite results to the Kamitake-Hibiki model for different shifts. The maturity of liabilities derived by the regulatory model is shorter than the maturity by the Kamitake-Hibiki model. Therefore, the decrease in liabilities cannot sufficiently offset the decrease in assets value. We should note that the impact of interest rate change on the banking book is dependent on the maturity structure of balance sheets. It is hard to achieve conservative risk management, based on the method which provides an estimate of excessively short maturity. It is important to construct the sophisticated core deposit model and to estimate a practical maturity appropriately, according to the behavior of depositors and the market environment.

## 4. CONCLUSION

In this paper, we do the dynamic simulation of an entire balance sheet of banks, and conduct the various sensitivity analyses to clarify the relationship between an economic value approach and modified earnings-based approach, both of which are utilized to manage interest risk in the banking book. The modified earnings-based approach can evaluate risk more comprehensively than the economic value approach by considering pass-through rates and future contracts. From the results, we find that these are sources of the profitability of banks, and it is important to build these factors into the model for risk management in the banking book.

Future issues are mainly as follows. First, future scenarios give a great impact on risk management in the modified earnings-based approach. We should discuss the method of generating future scenarios with respect to the behavior of depositors in the upward phase of interest rates. But it is difficult to estimate the parameters from historical data, because we have no more than one upward situation after April 2002 when the payoff system started on fixed deposits. This problem makes the interest rate risk management hard. We need to modify the model, based on the experiences occurred on the upward phase of interest rate.

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