# Multi-period ALM Optimization Model for a Household

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**Abstract**

Asset and liability management tools can be used for giving an financial advice to households or individual investors. We describe a multiperiod optimization model to determine an optimal set of asset mix, life insurance and fire insurance in conjunction with their life cycle and characteristics. Using three kinds of financial products, we can hedge risk against the death of the householder, the fire of the house, and inflation. The simulated path approach can be used to solve this problem. The household examples are illustrated to examine the usefulness of the model served as the financial consulting tools.

## 1 Introduction

We discuss an optimization model to obtain an optimal investment and insurance strategy for a household. Recently, financial institutions have promoted giving an financial advice for individual investors. How much will the household need to save when the householder retires? What kind of financial products should be purchased to hedge various risk such as market risk, inflation risk, and catastrophe insurance risk? Financial institutions need to recommend appropriate financial products to answer these questions in conjunction with a life cycle, current asset, and future income.

We clarify how a set of asset mix, life insurance, and fire insurance affect asset and liability management for the household. We develop the multiperiod optimization model with a set of financial products to hedge risk associated with the life cycle of the household and to save for the old age, and the simulated path approach (Hibiki, 2001b) can be used to solve this problem.

First studies in the literature for individual optimal investment strategy are Merton (1969) and Samuelson (1969) which proposed lifetime portfolio selection models. Merton (1971) extended his study, and proposed a model with a general utility function. The model determines the optimal asset mix and consumption in the planning period so that the utility of wealth can be maximized. Bodie, Merton and Samuelson (1992) extended the Merton problem (1971), and modeled the life cycle. They develop the lifetime model in consideration of the fact that the human capital (the present value of future income) changes into the real asset as it passes through age, and obtain the optimal investment strategy and consumption. Bodie and Crane(1997) analyze the relationship between the individual attribution(characteristics) and the stock holding

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ratio in the U.S.A. Yoshida, Yamada and Hibiki(2002) solve an optimal asset allocation problem for a household using multi-period optimization approach. Almost of these previous researches do not consider the cashflow associated with the insurance. In this paper, we develop the practical life cycle model involving the life insurance of a householder and the fire and casualty insurance.

This paper is organized as follows. Section 2 shows the current status of the individual preference for financial products. In Section 3, we define the household and three kinds of financial products such as securities, life insurance, and fire insurance to describe the model structure, and we introduce the concept of the simulated path approach. Section 4 presents the formulation for a multi-period optimization. We demonstrate some numerical tests for a hypothetical household in Section 5 and illustrate consulting three types of households in Section 6. Section 7 provides our concluding remarks.

## 2 Status of Holding of Financial Assets by Households

Table 1 shows "Public Opinion Survey on Household Financial Assets and Liabilities (2003)" by the Central Council for Financial Services Information.

Table 1: Breakdown of Financial Assets by Type of Financial Product

| **The average amount of financial assets** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Year | 1980 | 1985 | 1990 | 1995 | 2000 | 2001 | 2002 | 2003 |
| Total (In 10,000 Yen) | 482 | 688 | 1,181 | 1,287 | 1,448 | 1,439 | 1,422 | 1,460 |
| Deposits and savings | 321 | 403 | 549 | 693 | 807 | 837 | 829 | 912 |
| Money trusts and/or loan trusts | 23 | 44 | 65 | 70 | 39 | 30 | 24 | 19 |
| Life insurance and/or postal life insurance | 76 | 115 | 229 | 258 | 300 | 291 | 277 | 260 |
| Fire and casualty insurance | | | 21 | 24 | 33 | 31 | 38 | 30 |
| Personal annuity insurance | 7 | 13 | 32 | 50 | 70 | 66 | 69 | 65 |
| Bonds | 11 | 20 | 33 | 28 | 19 | 17 | 23 | 21 |
| Stocks | 27 | 49 | 125 | 90 | 103 | 90 | 94 | 96 |
| Investment trusts | 4 | 12 | 33 | 27 | 32 | 26 | 30 | 22 |
| Workers' property accumulation savings | 13 | 22 | 33 | 41 | 40 | 42 | 32 | 31 |
| Other financial products | - | 10 | 61 | 6 | 5 | 9 | 6 | 4 |
| **The average percentage of financial assets** | | | | | | | | |
| Year | 1980 | 1985 | 1990 | 1995 | 2000 | 2001 | 2002 | 2003 |
| Total | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
| Deposits and savings | 66.6% | 58.6% | 46.5% | 53.8% | 55.7% | 58.2% | 58.3% | 62.5% |
| Money trusts and/or loan trusts | 4.8% | 6.4% | 5.5% | 5.4% | 2.7% | 2.1% | 1.7% | 1.3% |
| Life insurance and/or postal life insurance | 15.8% | 16.7% | 19.4% | 20.0% | 20.7% | 20.2% | 19.5% | 17.8% |
| Fire and casualty insurance | 0.0% | 0.0% | 1.8% | 1.9% | 2.3% | 2.2% | 2.7% | 2.1% |
| Personal annuity insurance | 1.5% | 1.9% | 2.7% | 3.9% | 4.8% | 4.6% | 4.9% | 4.5% |
| Bonds | 2.3% | 2.9% | 2.8% | 2.2% | 1.3% | 1.2% | 1.6% | 1.4% |
| Stocks | 5.6% | 7.1% | 10.6% | 7.0% | 7.1% | 6.3% | 6.6% | 6.6% |
| Investment trusts | 0.8% | 1.7% | 2.8% | 2.1% | 2.2% | 1.8% | 2.1% | 1.5% |
| Workers' property accumulation savings | 2.7% | 3.2% | 2.8% | 3.2% | 2.8% | 2.9% | 2.3% | 2.1% |
| Other financial products | - | 1.5% | 5.2% | 0.5% | 0.3% | 0.6% | 0.4% | 0.3% |

The average amount of financial assets per household in 2003 stands at 14,600,000 yen, up

380,000 yen from last year. By type of financial product, deposits and savings constitutes the largest weight (62.5%) among all financial products, and subsequently total insurance reached 24.4%. The percentage of securities in total (bonds, stocks and investment trust) is only 9.5%. We examine the component percentages of financial products in time series. Deposits and savings keep constituting the largest percentage among all financial products. The percentage showed a decline till 1990, but it was continuing to rise afterwards, and reached 62.5% in 2003. The percentage of life insurance and fire and casualty insurance is not changing a lot, while personal annuity insurance increased recently compared with 1980. The percentage of stocks had its peak in 1990 caused by bubble economy, but it was seldom changing. We conclude that the component percentages of financial products are not changing a lot compared with 1980, in spite of the increase of the average amount of financial assets.

Figure 1 shows the average percentages of financial assets by age bracket of households.

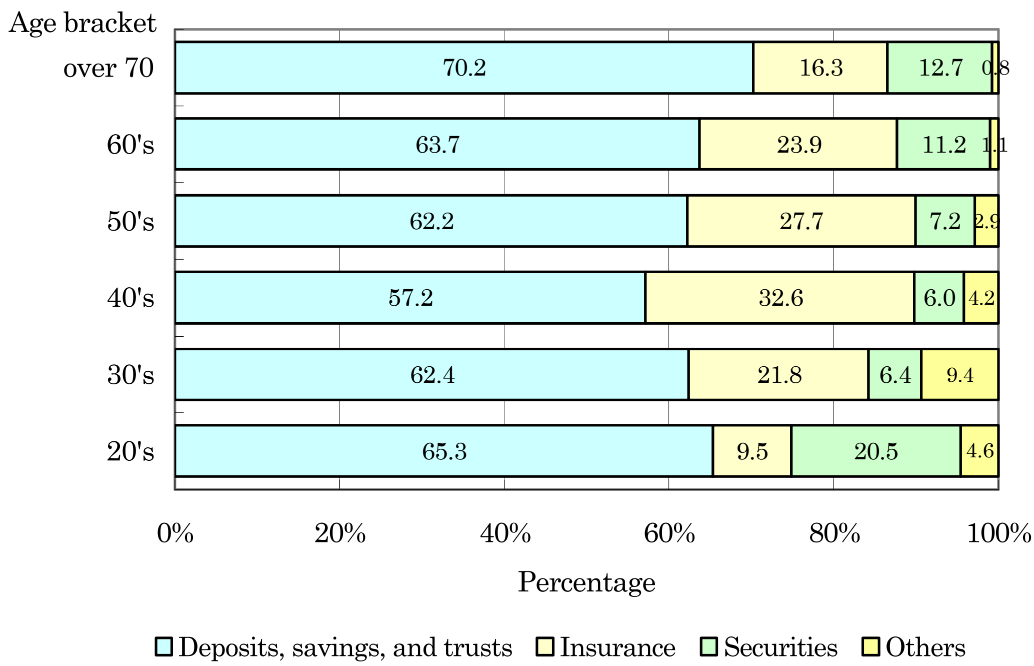


Figure 1: The average percentages of financial assets by age bracket of households

The percentage of deposits, savings and trusts is the highest in all of the age bracket, and it is more than 70% for 'over 70' especially. The 40's have the highest percentage of insurance and the lowest percentage of securities among all of the age bracket. In the age bracket of 'over 40's', the higher the age is, the less the percentage of insurance is, and the more the percentage of securities is.

Others in the 30's(9.4 %) is mainly constituted by workers' property accumulation savings. The system is used for savings of labors from the 20's to the 50's. The percentages of financial assets are different by age bracket. However, the characteristics of asset composition is similar through all ages, due to the extremely high percentage of deposits and savings. The portfolio of financial assets for households in Japan is not changing a lot over time, and by age bracket. This means that we need a financial advice for individuals, and individual investors (householders) should tailor their investment and insurance strategy to their life cycle.

Recently, Japanese financial institutions make strong efforts to the asset management for individuals. What kind of financial products should be held to hedge various risk ? We make a mathematical model to solve this problem. We illustrate practical examples to examine the usefulness of the model and to highlight the significance of the consulting service.

# 3 Model structure

## 3.1 Setting

**(1) Household**

We define a household as a group composed of a householder and members of family in this paper. Asset at time $t$ held by a household can be divided into two kinds of assets: the amount of financial asset $W_{1,t}$ and the amount of non-financial asset $W_{2,t}$. Income at time $t$ is a wage by the householder $m_t$ and investment return from the financial asset. There assumes to be two kinds of expenses: the living expenses $C_{1,t}$ and the purchase of non-financial assets $C_{2,t}$, such as a house, goods, and repair costs. The household is exposed to risk associated with two kinds of accidents: the death of the householder and the fire of the house. It is assumed that the death of the householder makes wage earnings stop, and the fire of the house damages a fraction $\alpha$ of non-financial assets. The household can purchase the life insurance and the fire insurance to hedge risk in addition to the investment of securities such as stocks and bonds.

We define the earning function of the householder $m_t$ and the cost functions of the household $C_{1,t}$, $C_{2,t}$ exogenously as time-dependent functions. We can obtain the optimal strategy in conjunction with the current state and the future plan of the household, such as the type of job of the householder, the family structure, the purchase plan of the house. The fraction of damage $\alpha$ caused by the fire disaster is assumed to be constant.

**(2) Objective function**

We assume that the current time is $0(t = 0)$, and a householder retires at time $T$, which is a planning horizon. The objective is the minimization or maximization of the function defined using the terminal amount of financial asset $W_{1,T}$. We select the financial products using two kinds of risk measures: the first-order lower partial moment, and conditional value at risk.

① First-order lower partial moment

The target of the financial asset $W_G$ is defined as the minimum level after the householder's retirement. The objective function $q(W_{1,T})$ is described as the expected amount of shortfall below target $W_G$.

$$q(W_{1,T}) = \mathrm{E}\left[|W_{1,T} - W_G|_-\right] \tag{1}$$

where $|a|_- = \max(-a, 0)$.

② Conditional value at risk

The objective function is defined as the expected amount of the financial asset on the condition that the amount of financial asset is under $\beta$-VaR($\equiv V_\beta$) where $\beta$ is any specified probability level (ex. $\beta = 0.95$), and it is maximized [1] .

$$\mathrm{CVaR}_\beta = V_\beta - \frac{1}{1-\beta} \cdot \mathrm{E}\left[ |W_{1,T} - V_\beta|_- \right] \tag{2}$$

**(3) Securities**

The investment in risky assets contributes to a hedge against inflation. We invest in $n$ risky assets and cash. A rate of return $R_{jt}$ of risky asset $j$ at time $t$ is calculated using the price $\rho_{jt}$ as follows.

$$R_{jt} = \frac{\rho_{jt}}{\rho_{j,t-1}} - 1, \ (t = 1, \ldots, T) \tag{3}$$

A risk-free rate $r_t$ at time $t(= 0, 1, \ldots, T-1)$ is fixed in the period from time $t$ to time $t+1$. We can assume any probability distributions of $R_{jt}$ and $r_t$ in the simulated path approach if we can sample random path for $R_{jt}$ and $r_t$. However, it is assumed that $R$ is normally distributed with mean vector $\boldsymbol{\mu}$, and the covariance matrix $\Sigma$ ($R \sim N(\boldsymbol{\mu}, \Sigma)$, and $r_t$ is constant for all $t$. We calculate the price $\rho_{jt}$ by using $R_{jt}$.

**(4) Life insurance**

Life insurance is the insurance against death for a householder with maturity $T$. If a householder purchases the life insurance and dies until time $T$, the household can receive the insurance money. In this model, we look upon the life insurance as the financial product which can hedge risk associated with the wage income earned by the householder. When the insurance policy is designed, pure premium should be determined so that the present value of future premium income can be equal to the present value of future premium payment. It is called the principle of equalization of income and expenditure. The yield used in calculating the present value is called the guaranteed interest rate.

Using the principle of equalization of income and expenditure, the relationship between a unit of present value of premium income and the corresponding insurance money $\theta_1$ is shown as :

$$1 = \sum_{t=1}^{T} \frac{\theta_1 \lambda_{1,t}}{(1+g_1)^t}, \ \text{or} \quad \theta_1 = \left\{ \sum_{t=1}^{T} \frac{\lambda_{1,t}}{(1+g_1)^t} \right\}^{-1} \tag{4}$$

where $g_1$ is the guaranteed interest rate of the insurance against death with maturity $T$, and $\lambda_{1,t}$ is the mortality rate at time $t$, or the probability that the person who is alive at time 0 will die at time $t$.

We can select single payment or level payment when we pay the life insurance premium. Premium of single payment per unit $y_{f_1}$ is equal to a unit of the present value of future premium income.

$$y_{f_1} = 1 \tag{5}$$

Premium of level payment per unit $y_{f_2}$ is calculated as follows, because only insured person who is alive pays premium.

[1] We can also define the CVaR of the initial amount minus the terminal amount of financial asset instead of the amount of financial asset. This gives the minimization of the objective function. The equivalent optimal solution can be derived even if we change the definition of CVaR.

$$y_{f_2} = \left[ \sum_{t=0}^{T-1} \left\{ \frac{1 - \sum_{i=0}^{t} \lambda_{1,i}}{(1+g_1)^t} \right\} \right]^{-1} \tag{6}$$

**(5)Fire insurance**

The household purchases one year fire insurance to hedge the damage of non-financial asset due to the fire. The household can update the insurance contract every year, and purchase the fire insurance policy corresponding to the future non-financial asset. Using the principle of equalization of income and expenditure, the relationship between one unit of present value of premium income and the corresponding insurance money $\theta_2$ is shown as :

$$1 = \frac{\theta_2 \lambda_2}{1+g_2}, \quad \text{or} \quad \theta_2 = \frac{1+g_2}{\lambda_2} \tag{7}$$

where $g_2$ is the guaranteed interest rate of the one year fire insurance, and $\lambda_2$ is the rate of the fire, or the probability that the fire occurs. It is independent on time $t$.

We can only select single payment because of one year fire insurance. Premium of single payment per unit $y_F$ is equal to a unit of the present value of future premium income.

$$y_F = 1 \tag{8}$$

**(6) Purchasing a house**

We illustrate the case that we take into consideration of buying a house and we consult a household in Section 6. We explain the relationship between cash flow of purchasing a house and the change of asset value.

We assume that a household purchases a house with a down payment and a debt loan from banks ($H_t$). The debt loan $H_t$ is the difference between the price of the house and the down payment. The debt loan $H_t$ is the cash inflow, and the consumption expenditures for non-financial asset $C_{2,t}$ is the cash outflow. However, the total amount of non-financial asset, $W_{2,t}$ is increased by expenditures $C_{2,t}$ at time $t$ when the house is purchased. the household has to pay the debt loan periodically under the determined interest rate and the loan period after the householder bought a house. In this paper, we include periodic payments in the expenditures for life ($C_{1,t}$).

## 3.2 Simulated path approach

It is critical for stochastic modeling to handle uncertainties and investment decisions appropriately. The decisions have to be independent from knowledge of actual paths that will occur. Thus, we must define a set of decision variables and a set of constraints to prevent an optimization model from being solved by anticipating events in the future. In addition, we need a sufficient number of paths to get a better accuracy with respect to the future possible events.

The concept of scenarios is typically employed for modeling random parameters in multi-period stochastic programming models. Scenarios are constructed via a tree structure as in the left-hand-side of Figure 2 (see Mulvey and Ziemba, 1995 and 1998 for a detailed discussion). The model is based on the expansion of the decision space, taking into account a conditional nature of the scenario tree. Conditional decisions are made at each node, subject to the modeling constraints. To ensure that the constructed representative set of scenarios covers the set of possibilities to a sufficient degree, the numbers of decision variables and constraints in the scenario tree may grow exponentially. This model is called a scenario tree model.

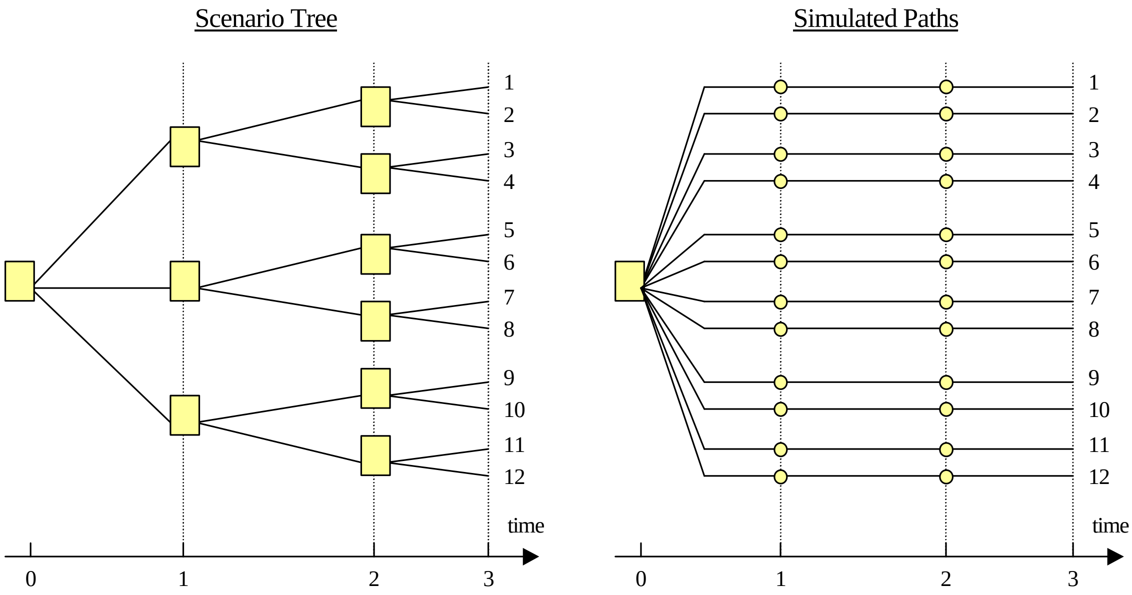


Figure 2: A scenario tree and simulated paths

Meanwhile, simulated paths give another description of scenarios shown as in the right-hand-side of Figure 2. Hibiki [2000, 2001b] developed a simulated path model in a multi-period optimization framework. If we formulate stochastic differential equations or time series models associated with asset returns, discrete asset returns are generated by a standard Monte Carlo simulation technique to describe uncertainties more accurately than would the scenario tree as in the left-hand-side of Figure 2. However, if a decision is made on the associated path, the model is solved anticipating the event in the future. Therefore, the rule that the same investment decision is made at each time is defined to satisfy the non-anticipativity condition in the simulated path model.

We do not use the scenario tree model, but the simulated path model. This is because the model involves determining the optimal life and fire insurance money, and we need a lot of paths at each time to describe the mortality rate and the rate of the fire. According to the life insurance standard life table(1996), the mortality rate for 50 years old men is 0.379%. Even if the householder dies on a path to describe the appropriate mortality rate, we need 264 paths at each time. If we construct a scenario tree over 30 years, we have to generate an enormous tree, and we cannot solve the problem in practice. Therefore, it is essential for this type of the problem to use the simulated path approach, and we use the simulated path model in this paper [2] . The formulation and numerical tests with the hybrid model are our future research.

[2] Hibiki(2001c, 2003) developed the hybrid model, which not only describes the uncertainties on the simulated

# 4 Optimization Model

We formulate and solve a multi-period optimization model with simulated path structure, and calculate an optimal asset mix and insurance money of a household.

## 4.1 Notations

**(1) Subscript/Superscript**

$j$ : asset ($j = 1, \ldots, n$).

$t$ : time ($t = 1, \ldots, T$).

$i$ : path ($i = 1, \ldots, I$).

**(2) Parameters**

$I$ : number of simulated paths.

$\rho_{j0}$ : price of risky asset $j$ at time 0, ($j = 1, \ldots, n$).

$\rho_{jt}^{(i)}$ : price of risky asset $j$ of path $i$ at time $t$, ($j = 1, \ldots, n;\ t = 1, \ldots, T;\ i = 1, \ldots, I$).

$$\rho_{j1}^{(i)} = \left(1 + R_{j1}^{(i)}\right) \rho_{j0}, \quad (j = 1, \ldots, n;\ i = 1, \ldots, I)$$

$$\rho_{jt}^{(i)} = \left(1 + R_{jt}^{(i)}\right) \rho_{j,t-1}^{(i)}, \quad (j = 1, \ldots, n;\ t = 2, \ldots, T;\ i = 1, \ldots, I)$$

where $R_{jt}^{(i)}$ is the rate of return of risky asset $j$ on path $i$ at time $t$.

$r_0$ : interest rate in period 1, (the rate at time 0).

$r_{t-1}^{(i)}$ : interest rate in period $t$ (the rate of path $i$ at time $t-1$), ($t = 2, \ldots, T;\ i = 1, \ldots, I$).

$\tau_{1,t}^{(i)}$ : one if a householder dies on path $i$ at time $t$ and zero if otherwise.

$\tau_{2,t}^{(i)}$ : one if the fire of the house occurs and zero if it does not occur.

$\tau_{3,t}^{(i)}$ : one if a householder is alive on path $i$ at time $t$ and zero if otherwise.

$\lambda_{1,t}$ : mortality rate at time $t$ : $\lambda_{1,t} = \Pr(\tau_{1,t} = 1) = \dfrac{1}{I} \displaystyle\sum_{i=1}^{I} \tau_{1,t}^{(i)}$

$\lambda_2$ : rate of the fire(which is time independent) : $\lambda_2 = \Pr(\tau_{2,t} = 1) = \dfrac{1}{I} \displaystyle\sum_{i=1}^{I} \tau_{2,t}^{(i)}$

$g_1$ : guaranteed interest rate on life insurance policies.

$f_1$ : one if single payment life insurance is bought and zero if level payment life insurance is bought.

$y_{f_1}$ : premium of single payment life insurance per unit: $y_{f_1} = 1$
(A unit of insurance policy corresponds to the present premium of 1 yen.)

$y_{f_2}$ : premium of level payment life insurance per unit derived by Equation (6).

$y_{L,t}^{(i)}$ : premium of life insurance per unit at time $t$:
$y_{L,t}^{(i)} = y_{f_1} \cdot f_1 \tau_{4,t} + y_{f_2} \cdot (1 - f_1) \tau_{3,t}^{(i)}$, where $\tau_{4,0} = 1$, $\tau_{4,t} = 0 (t \neq 0)$.

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path structure but also makes conditional decisions on the tree structure. The hybrid model is allowed to expand the decision space and to make conditional decisions as in the scenario tree model. The simulated path model is a special version of the hybrid model.

$\theta_1$ : life insurance money per unit derived by Equation (4).

$L_t^{(i)}$ : life insurance money per unit on path $i$ at time $t$: $\quad L_t^{(i)} = \tau_{1,t}^{(i)}\theta_1$

$g_2$ : guaranteed interest rate on fire insurance policies.

$y_F$ : premium of fire insurance per unit : $y_F = 1$

$\theta_2$ : one year fire insurance money per unit derived by Equation (7).

$F_t^{(i)}$ : one year fire insurance money per unit on path $i$ at time $t$ : $\quad F_t^{(i)} = \tau_{2,t}^{(i)}\theta_2$

$\alpha$ : loss ratio of non-financial asset due to the fire of the house.

$A_t^{(i)}$ : loss of non-financial asset due to the fire of the house on path $i$ at time $t$:

$$A_t^{(i)} = \tau_{2,t}^{(i)} \cdot \alpha \cdot (1-\gamma) \cdot W_{2,t-1}^{(i)}$$

$m_t^{(i)}$ : wage on path $i$ at time $t$.

$M_t^{(i)}$ : wage income a householder earns on path $i$ at time $t$: $\quad M_t^{(i)} = \tau_{3,t}^{(i)} m_t^{(i)}$

$H_t^{(i)}$ : debt loan on path $i$ at time $t$.

$C_{1,t}^{(i)}$ : consumption expenditures for life on path $i$ at time $t$.

$C_{2,t}^{(i)}$ : consumption expenditures for non-financial asset on path $i$ at time $t$.

$C_t^{(i)}$ : total consumption expenditures on path $i$ at time $t$: $\quad C_t^{(i)} = C_{1,t}^{(i)} + C_{2,t}^{(i)}$

$\gamma$ : depreciation ratio of non-financial asset.

$W_{1,t}^{(i)}$ : total amount of financial asset on path $i$ at time $t$.
($W_{1,0}$ is a total amount at time 0.)

$W_{2,t}^{(i)}$ : total amount of non-financial asset on path $i$ at time $t$ [3] : $\quad W_{2,t}^{(i)} = (1-\gamma) \cdot W_{2,t-1}^{(i)} + C_{2,t}^{(i)}$

$W_E$ : lower bound of expected terminal amount of financial asset.

$W_G$ : target terminal amount of financial asset used in the LPM formulation.

$\beta$ : Probability level used in the CvaR formulation.

$L_{v,t}$ : lower bound of cash at time $t$. When $L_{v,t} < 0$, the borrowing can be allowed.

**(3) Decision variables**

$z_{jt}$ : investment unit of asset $j$ at time $t$. $(j = 1, \dots, n;\ t = 0, \dots, T-1)$

$v_0$ : cash at time 0

$v_t^{(i)}$ : cash of path $i$ at time $t$. $(t = 1, \dots, T-1)$

$u_L$ : number of life insurance bought at time 0.

$u_{F,t}$ : number of one-year fire insurance bought at time $t$.

$V_\beta$ : $\beta$-VaR used in the CVaR formulation.

$q^{(i)}$ : ① (LPM) shortfall below target terminal amount of financial asset($\equiv W_G$) on path $i$,

$$q^{(i)} \equiv \max\left(W_G - W_{1,T}^{(i)}, 0\right)$$

[3] Non-financial asset is decreased by $A_t^i$ due to the fire, but it is increased by $A_t^i$ by spending the same money to recover the loss. Therefore, $A_t^i$ does not affect the total amount of non-financial asset. Instead, it affects the cash flow as shown in Equation (9) and (10).

② (CVaR) shortfall below $\beta$-VaR($\equiv V_\beta$) of terminal amount of financial asset($\equiv W_{1,T}^{(i)}$) on path $i$, $\quad q^{(i)} \equiv \max\left(V_\beta - W_{1,T}^{(i)}, 0\right)$

## 4.2 Objective function, return requirement and cash flow except trading asset

### (1) Objective function

The objective is the minimization of the first-order lower partial moment $\mathrm{LPM}_1$ or the maximization of the CVaR associated with terminal amount of financial asset subject to the minimum return requirement. Two kinds of risk measures are defined as follows.

① First-order lower partial moment

$$\mathrm{LPM}_1 = \mathbf{Min}\left\{ \frac{1}{I}\sum_{i=1}^{I} q(i) \middle| W_{1,T}^{(i)} + q^{(i)} \geq W_G,\ (i = 1, \ldots, I) \right\}$$

② Conditional value at risk

$$\mathrm{CVaR}_\beta = \mathbf{Max}\left\{ V_\beta - \frac{1}{(1-\beta)I}\sum_{i=1}^{I} q(i) \middle| W_{1,T}^{(i)} - V_\beta + q^{(i)} \geq 0,\ (i = 1, \ldots, I) \right\}$$

Even if CVaR of $W_0 - W_{1,T}^{(i)}$ is used to minimize the objective, we have the same solutions as the solutions derived from the maximization of CVaR of $W_{1,T}^{(i)}$.

### (2) Return requirement

We define the expected terminal amount of financial asset $\mathrm{E}[W_{1,T}]$ as return measure. The lower bound is $W_E$, and therefore the minimum return requirement is formulated as :

$$\sum_{i=1}^{I} W_{1,T}^{(i)} \geq W_E$$

### (3) Cash flow except trading assets

Cash flow constraints are important in the multi-period optimization approach. Cash flow except trading assets $D_t^{(i)}$ is associated with wages, expenditures, and insurance. It is formulated as follows.

① $t = 1, \cdots, T-1$

$$\begin{aligned} D_t^{(i)} &= M_t^{(i)} + H_t^{(i)} - C_t^{(i)} - y_{L,t}^{(i)} u_L - y_F u_{F,t} + L_t^{(i)} u_L + F_t^{(i)} u_{F,t-1} - A_t^{(i)} \\ &= \tau_{3,t}^{(i)} m_t^{(i)} + H_t^{(i)} - \left(C_{1,t}^{(i)} + C_{2,t}^{(i)}\right) - y_{L,t}^{(i)} u_L - y_F u_{F,t} + \tau_{1,t}^{(i)}\theta_1 u_L + \tau_{2,t}^{(i)}\theta_2 u_{F,t-1} \\ &\quad -\tau_{2,t}^{(i)}(1-\gamma)W_{2,t-1}^{(i)}\alpha \end{aligned} \tag{9}$$

② $t = T$ : Insurance payment is not required at time $T$.

$$\begin{aligned} D_T^{(i)} &= M_T^{(i)} + H_T^{(i)} - C_T^{(i)} + L_T^{(i)} u_L + F_T^{(i)} u_{F,T-1} - A_T^{(i)} \\ &= \tau_{3,T}^{(i)} m_T^{(i)} + H_T^{(i)} - \left(C_{1,T}^{(i)} + C_{2,T}^{(i)}\right) + \tau_{1,T}^{(i)}\theta_1 u_L + \tau_{2,T}^{(i)}\theta_2 u_{F,T-1} \\ &\quad -\tau_{2,T}^{(i)}(1-\gamma)W_{2,T-1}^{(i)}\alpha \end{aligned} \tag{10}$$

### 4.3 Formulation

**(1) First-order lower partial moment**

$$\textbf{Minimize} \quad \frac{1}{I}\sum_{i=1}^{I} q^{(i)} \tag{11}$$

**subject to**

$$\sum_{j=1}^{n} \rho_{j0} z_{j0} + v_0 + y_{L,0} u_L + y_F u_{F,0} = W_{1,0} \tag{12}$$

$$(W_{1,1}^{(i)} =) \ \sum_{j=1}^{n} \rho_{j1}^{(i)} z_{j0} + (1+r_0) v_0 + D_1^{(i)} = \sum_{j=1}^{n} \rho_{j1}^{(i)} z_{j1} + v_1^{(i)}, \quad (i = 1, \ldots, I) \tag{13}$$

$$(W_{1,t}^{(i)} =) \ \sum_{j=1}^{n} \rho_{jt}^{(i)} z_{j,t-1} + \left(1 + r_{t-1}^{(i)}\right) v_{t-1}^{(i)} + D_t^{(i)} = \sum_{j=1}^{n} \rho_{jt}^{(i)} z_{jt} + v_t^{(i)},$$
$$(t = 2, \ldots, T-1;\ i = 1, \ldots, I) \tag{14}$$

$$W_{1,T}^{(i)} = \left\{ \sum_{j=1}^{n} \rho_{jT}^{(i)} z_{j,T-1} + \left(1 + r_{T-1}^{(i)}\right) v_{T-1}^{(i)} \right\} + D_T^{(i)},\ (i = 1, \ldots, I) \tag{15}$$

$$\frac{1}{I}\sum_{i=1}^{I} W_{1,T}^{(i)} \geq W_E \tag{16}$$

$$W_{1,T}^{(i)} + q^{(i)} \geq W_G,\ (i = 1, \ldots, I) \tag{17}$$

$$z_{jt} \geq 0, \quad (j = 1, \ldots, n;\ t = 0, \ldots, T-1)$$

$$v_0 \geq L_{v,0}$$

$$v_t^{(i)} \geq L_{v,t}, \quad (t = 1, \ldots, T-1;\ i = 1, \ldots, I)$$

$$u_L \geq 0$$

$$u_{F,t} \geq 0, \quad (t = 0, \ldots, T-1)$$

$$q^{(i)} \geq 0, \quad (i = 1, \ldots, I)$$

where $W_{1,t}^{(i)}$ is an amount of financial asset on path $i$ at time $t$.

**(2) Conditional value at risk**

Equation (11) is replaced with :

$$\textbf{Maximize} \quad V_\beta - \frac{1}{(1-\beta)I}\sum_{i=1}^{I} q^{(i)} \tag{18}$$

and Equation (17) is also replaced with :

$$W_{1,T}^{(i)} - V_\beta + q^{(i)} \geq 0,\ (i = 1, \ldots, I) \tag{19}$$

If $V_\beta$ is replaced with $W_G$, the formulation is equivalent to the formulation using the first-order lower partial moment.

# 5 Numerical examples

## 5.1 Preparation

We report some results of numerical test. The parameter values used in the examples are shown in Table 2. We show the result of the CVaR problem.

Table 2: Parameter values

| Parameter | Value |
|---|---|
| number of risky assets | $n = 1$ |
| length of one period | one year |
| retirement age of a householder | 60 years old |
| number of periods | $T = 60 -$ current age of a householder |
| expected rate of return of risky asset | $\mu = 0.1$ |
| standard deviation of rate of return of risky asset | $\sigma = 0.2$ |
| risk-free rate | $r = 0.03$ |
| mortality rate | $\lambda_{1,t}(\dagger)$ |
| rate of the fire | $\lambda_2 = 0.005$ |
| guaranteed rate on life insurance | $g_1 = 0.05$ |
| guaranteed rate on fire insurance | $g_2 = 0.05$ |
| payment of life insurance | level payment： $f_1 = 0$ |
| wage income（million yen） | $m_t = 0.125 \times t + 5$ |
| consumption expenditure（million yen）<br>(expenditure for purchasing non-financial asset) | $C_t = C_{1,t} + C_{2,t} = 0.125 \times t + 4.25$<br>$(C_{2,t} = 0.4 \times (1 + 0.01)^{t-1})$ |
| initial amount of financial asset（million yen） | $W_{1,0} = 10$ |
| amount of non-financial asset（million yen） | $W_{2,t} = 10 \times (1 + 0.01)^t$ |
| initial amount of non-financial asset（million yen） | $W_{2,0} = 10$ |
| depreciation rate of non-financial asset | $\gamma = 0.03$ |
| loss of non-financial asset due to the fire | $\alpha = 1$ |
| lower bound of cash (million yen) | $L_{v,0} = 0,\ L_{v,t} = -1,000(t \neq 0)$ |
| probability level | $\beta = 0.8$ |
| number of paths | $I = 5,000$ |

† The rates are estimated by the "life insurance standard life table 1996 for men.

## 5.2 Householder's age and optimal strategy

Lower bound of expected amount of financial asset is calculated by

$$W_E = W_{1,0} \times (1 + \mu_W)^T$$

where $\mu_W = 10\%$. Figure 3 shows the optimal investment amount of risky asset, the optimal life insurance money, and the optimal fire insurance money at time 0 for each householder's age. For example, when the householder is 50 years old, the optimal investment is 5.14 million yen, the optimal life insurance is 53.62 million yen, and the optimal fire insurance is 10.40 million yen.

The higher the householder's age is, the less the optimal life insurance money at time 0 is. Because a younger householder earns a large amount of wage income in the future, he needs to purchase a more expensive life insurance in order to hedge risk associated with the wage income. The optimal fire insurance money at time 0 is approximately constant (about 10 million yen) without reference to the householder's age, because initial amount of non-financial asset hedged at time 0 is assumed to be constant (10 million yen) and $\alpha = 1$ in this example. It is approximately equal to loss of the non-financial asset due to the fire. We obtain the practical result which shows that only the fire insurance can hedge loss due to the fire. The higher the householder's age is, the less the optimal amount of risky asset at time 0 is. This reason is that younger householders can take a larger risk for a longer investment period, and therefore invest in a larger amount of risky asset.

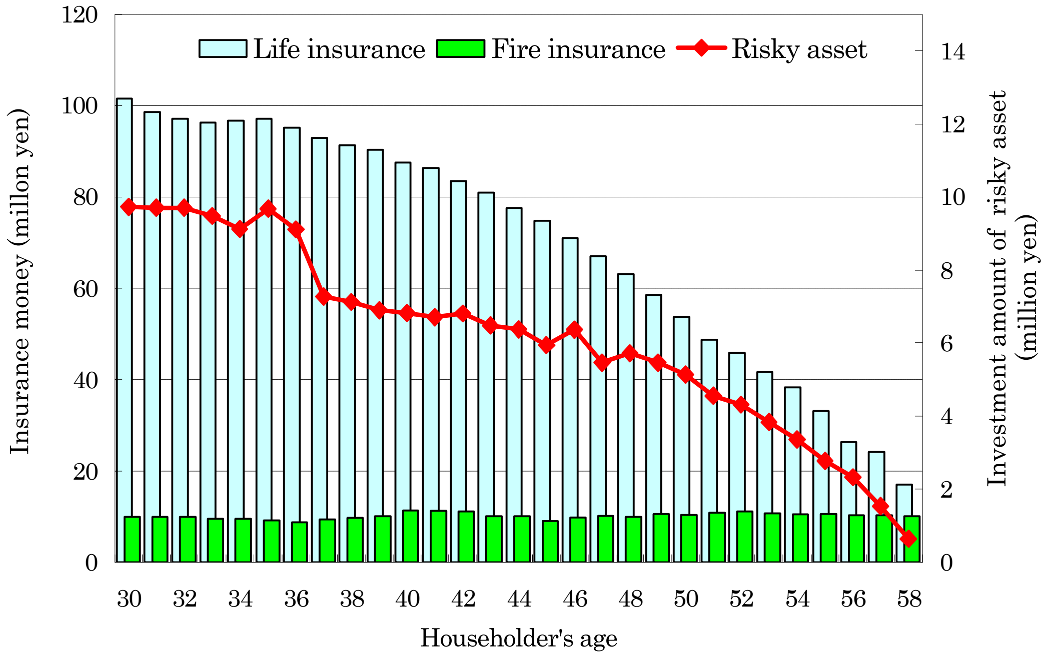


Figure 3: Householder's age and optimal strategy

### 5.3 Expected terminal amount of financial asset and optimal strategy

We examine the model using the various lower bound of the expected terminal amount of financial asset. The householder is 50 years old, and therefore $T = 10$. The value of $W_E$ is given every one million yen from 20 million yen through 30 million yen. We show the relationship between the expected terminal amount and the optimal strategy at time 0 in Figure 4.

The optimal life insurance money is approximately constant in the range from 50 million yen through 55 million yen without reference to the expected terminal amount of financial asset. The optimal fire insurance money is approximately 10 million yen. The higher the expected terminal amount of financial asset is, the more the optimal investment amount of risky asset is. This is because the investment in risky asset needs to be increased to achieve the higher expected return, and the financial asset can be purchased as a hedge against inflation, but it cannot be used to hedge risk associated with the life and the fire [4] .

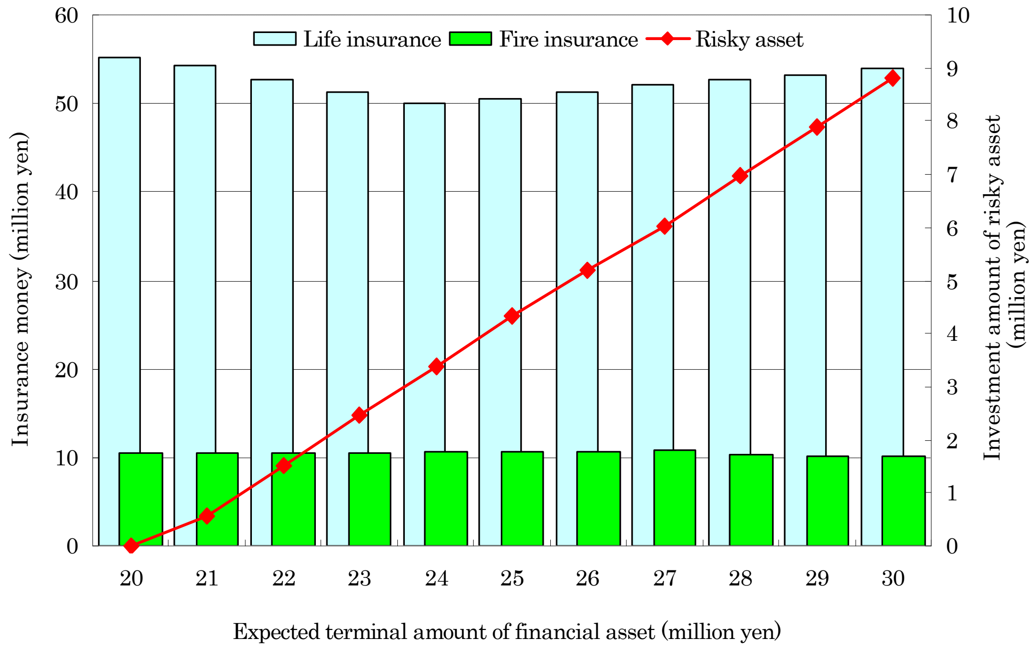


Figure 4: Expected terminal amount of financial asset and optimal strategy

Next, we show the optimal fire insurance from $t = 0$(present) through $t = T-1$(previous year of the retirement) for $W_E = 28$ million yen in the left-hand-side of Figure 5. The optimal fire insurance is approximately equal to the amount of non-financial asset. This is consistent with the result in Figures 3 and 4. We show the optimal investment ratio in the right-hand-side of Figure 5. As time passes, investment ratio of risky asset decreases from 72% to 57%. This is consistent with the result in Figure 3 that younger households tend to invest in more risky asset.

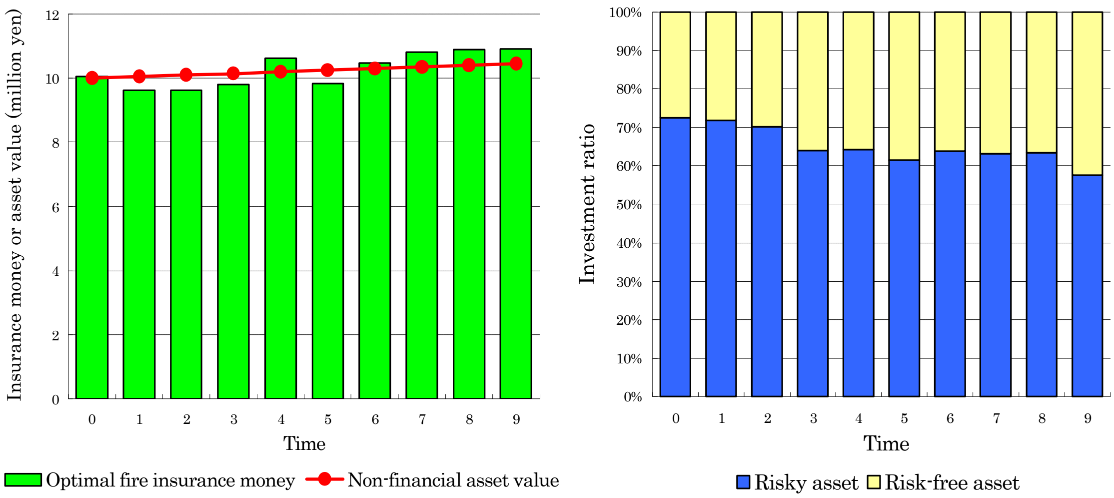


Figure 5: Optimal fire insurance money and investment ratio of risky asset

[4]Note that three kinds of financial products, the life insurance, the fire insurance and risky asset, are used to hedge risk against the death of the householder, the fire of the house, and inflation, respectively.

### 5.4 Optimal strategy for two risky assets case

We test an example of two risky assets. We substitute parameters in Table 3 for parameters associated with risky assets in Table 2.

Table 3: Parameter values

| Parameter | Value |
|---|---|
| number of risky assets | $n = 2$ |
| expected rate of return of risky asset A | $\mu_A = 0.07$ |
| standard deviation of rate of return of risky asset A | $\sigma_A = 0.1$ |
| expected rate of return of risky asset B | $\mu_B = 0.13$ |
| standard deviation of rate of return of risky asset B | $\sigma_B = 0.3$ |
| correlation coefficient between risky assets A and B | $\rho_{AB} = 0$ |

The householder is 50 years old, and therefore $T = 10$. The value of $W_E$ is given every one million yen from 20 million yen through 30 million yen. We show the relationship between the expected terminal amount and the optimal strategy in the left-hand-side of Figure 6.

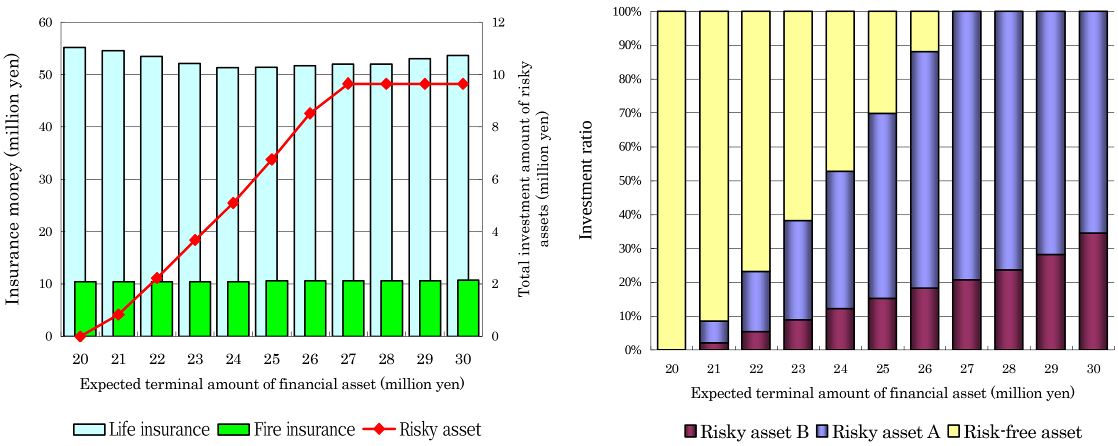


Figure 6: Expected terminal amount of financial asset and optimal strategy

We compare the left-hand-side of Figure 6 with Figure 4. The result shows that the optimal life insurance is about 55 million yen, and the optimal fire insurance is about 10 million yen without reference to the expected terminal amount of financial asset. We have almost the same result as one risky asset case. As the expected terminal amount increases, the optimal total investment in two risky assets is more than the optimal investment in one risky asset in Figure 4. The right-hand-side of Figure 6 shows the relationship between the expected terminal amount of financial asset and investment ratio. The higher the expected terminal amount of financial asset is, the more risky assets we invest in from $W_E = 20$ million yen through $W_E = 27$ million yen. We invest all money in risky assets over $W_E = 27$ million yen. The ratio of risky asset A is higher than riskier asset B. However, because risky asset B has a higher expected return, the percentage of asset B goes up and the percentage of asset A goes down as $W_E$ becomes large.

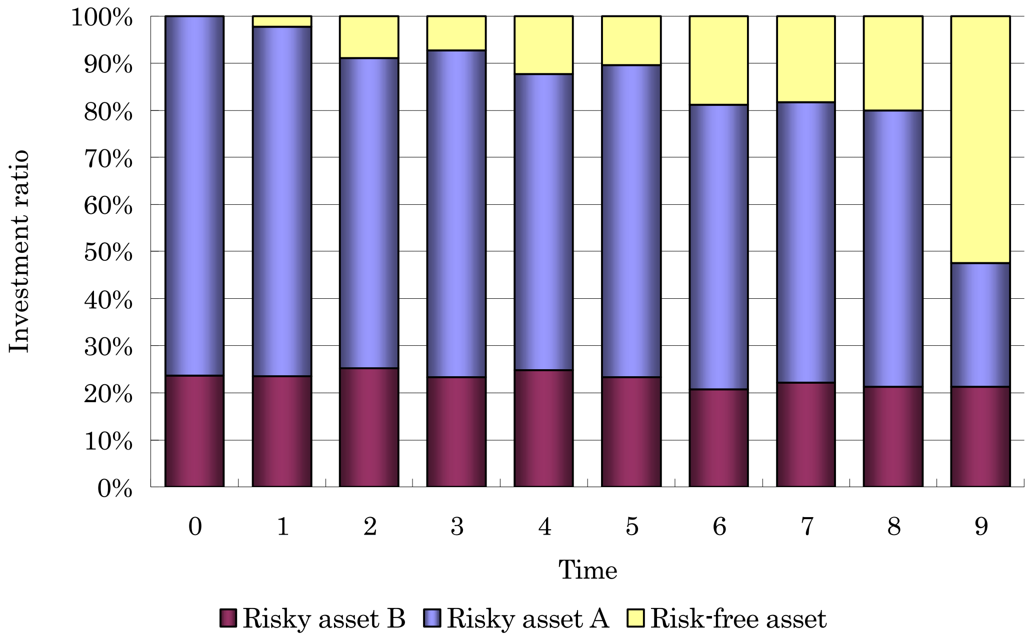


Figure 7: Optimal investment ratio from $t = 0$ through $t = T - 1$

Next, we show the optimal investment ratio from $t = 0$ through $t = T - 1$ for $W_E = 28$ million yen in Figure 7. Total investment ratio of risky assets is 100% at time 0(76.4% for asset A and 23.6% for asset B), however it declines to 79.9% at time 8 and 47.6% at time 9(26.3% for asset A and 21.4% for asset B).

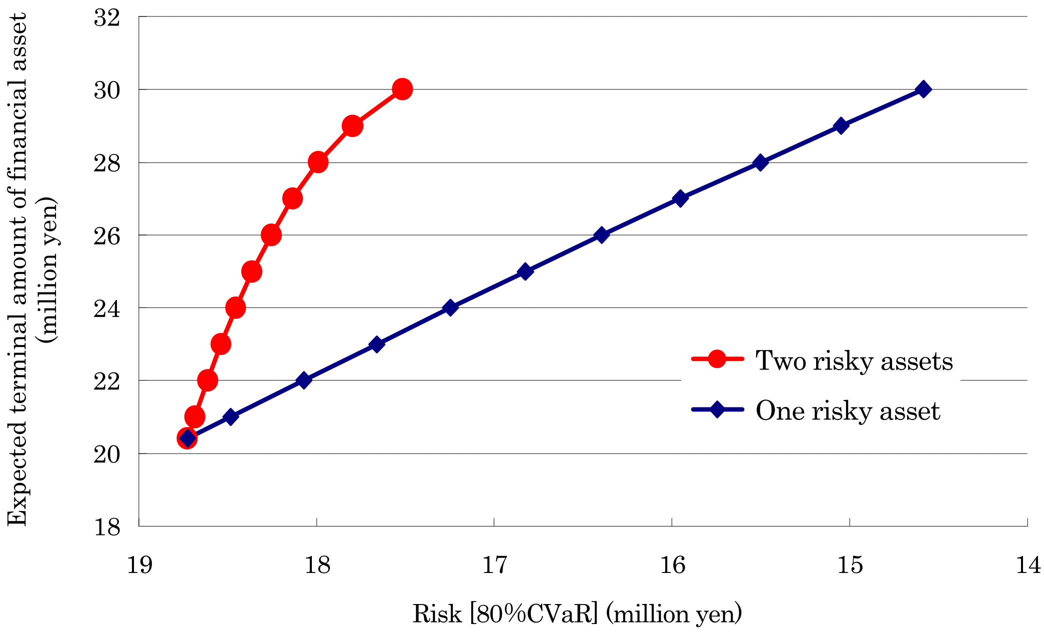


Figure 8: Efficient frontiers of the terminal amount of financial asset

Figure 8 shows the two efficient frontiers of the terminal amount of financial asset in the expected value and CVaR of $\beta = 0.8$ space. One is the efficient frontier derived with two risky assets which shape is concave, and the other is the efficient frontier derived with one risky asset which shape is a straight line. The frontier using two assets dominates the frontier using one asset. This graph clarifies the effect of diversification on portfolio risk.

# 6 Illustration of consulting households for practical use

Our goal is to develop an asset and liability management tool(a financial consulting tool) to give an financial advice to households or individual investors. We examine the model for the goal.

## 6.1 Assumption setting

Suppose that we give an financial advice to three households in Table 4. Three households have the same family structure and age. However, the occupation of the householders, the educational plan, and the purchase plan of a new house are different among them.

Table 4: Attribution of the households

| Item | | Household A | Household B | Household C |
|---|---|---|---|---|
| family structure (age) | householder | 30 years old | | |
| | spouse | 28 years old | | |
| | first child | 0 years old | | |
| | second child | — | birth in 3 years | — |
| occupation | householder | local government | financial institution | medical practitioner |
| | spouse | housewife | | |
| school (education) | elementary to senior high | public | private | private |
| | university (house) | national (living alone) | private (at home) | medical (at home) |
| wedding payment | | child | parents | child |
| current house | type | official residence | apartment | single house with a clinic |
| | rent | 25,000 yen | 200,000 yen | 420,000 yen |
| | area | local city | center of Tokyo | outskirts of Tokyo |
| new house buying plan | time | when householder'age is 40 years old | | |
| | type | single house | apartment | single house with a clinic |
| | price | 30 million yen | 50 million yen | 100 million yen |
| | initial payment | 10 million yen | 20 million yen | 30 million yen |
| | interest rate | 6% fixed | | |
| | period | 20 years | | |

The householder of Household A works at a local government, and the family lives in a relatively cheap official residence. However, the household plans that it will prepare 10 million yen as a down payment ten years later and buy a single house in a local city which costs 30 million yen. The parents make an educational plan that the child will go to the public elementary school, junior high school, and high school, and the national university. The householder of Household B works at a financial institution, and the household plans that it will prepare 20 million yen as a down payment ten years later and buy an apartment in the center of Tokyo which costs 50 million yen. The parents make an educational plan that the child will go to the private elementary school, junior high school, high school, and university. They will support the wedding payment for their children. The householder of Household C is a medical practitioner at his clinic. He wants his child to go to the medical school, and to take over his clinic. The household plans that it will prepare 30 million yen as a down payment ten years later and buy a single house with a clinic which costs 100 million yen.

When a financial planner starts to consult a household, the life plan of the household is examined by making a cash flow table from the present through the target time. The cash flow table is made so that it reflects the attribution of the household such as its income, expenditure, life event in the future.

**(1) Wage incomes and consumption expenditures**

We show the wage incomes and expenditures of three households in Figure 9. The wage income depends on the householder's age and his occupation. We calculate the wage income of the household over time based on the Census of wage by Ministry of Health, Labor and Welfare(2003) as in the left-hand-side of Figure 9. Household A whose householder is a local government employee has the wage income with the upward trend. The increasing rate in salary is constant without reference to age and therefore the wage income is the linear function of age. Household B whose householder is a financial institution employee has a highly increasing rate in salary until 50 years old, however the wage income declines afterwards. Household C whose householder is a medical practitioner has a constant wage income without reference to age, and earns the highest wage among them.

The consumption expenditure depends on the wage income, family structure and school(education) plan. We calculate average consumption expenditures with respect to each number of family and each income level of family based on the national survey of family income and expenditure(1999) by Statistic Bureau, Ministry of Internal Affairs and Communications. We calculate average educational expenses based on the survey of household expenditure on education per student(2001) , the survey of student life by Ministry of Education, Culture, Sports, Science and Technology. We show each consumption expenditure of households except a rent and a new house buying cost in the right-hand-side of Figure 9.

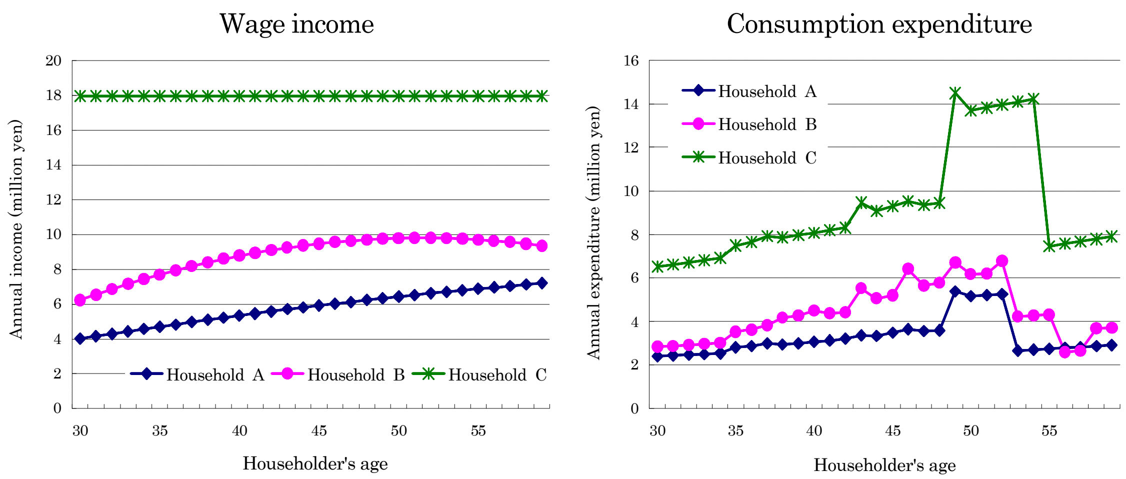


Figure 9: Annual wage income and consumption expenditure of each household

The higher the wage income is, the larger the consumption expenditure is. The consumption expenditure depends on the number of children and the educational plan. The difference of the annual expenditure between Household A and B is about 400 thousand yen in the beginning of lifetime. However, Household B needs more 2.77 million yen than Household A when the householders are 46 years old, and this is a maximum difference. This is because Household B has two children who will go to a private school, and Household A has a child who will go to a public school. Household C has to pay about 5 million yen annually for 6 years as the educational cost because the child will go to a medical school. The annual consumption expenditure of Household C in this period is about 14 million yen, and it is twice more than Households A and B.

**(2) Housing expenditure**

We divide the housing expenditure into three parts: rent (before purchasing the house), initial payment (down payment when purchasing the house), and annual payment (after purchasing the house). The households continue to pay annual rent in Table 4 until they purchase their own house. The initial payment is paid when the households are 40 years old and they purchase their own house. Households A, B, and C pay 10, 20, and 30 million yen respectively. Annual payment assumes to be equal payment with interest, and it is calculated with loan account (house price minus initial payment), loan period, and interest rate. Household A, B, and C amortize their debts of 20, 30, and 50 million yen over 20 years at 6% interest, and therefore annual payments are to be 1.74, 2.62, and 6.10 million yen, respectively.

Which should three kinds of housing expenditures be included in the expenditure for life $C_{1,t}$ or non-financial asset $C_{2,t}$ ? Rent is the expenditure for life $C_{1,t}$ because it does not contribute to increase the non-financial asset. We assume that a fraction($\alpha$) of non-financial asset ($W_{2,t}$) is lost when the fire occurs, but it is kept the same value by spending money and restoring the non-financial asset. The house is the non-financial asset of the household, and the loss due to the fire is not a fraction of total loan paid before the fire occurs, but a fraction of the house price. The initial and annual payments are not included in the expenditure of non-financial asset $C_{2,t}$, but the expenditure for life $C_{1,t}$, because the house price is added to the non-financial asset when the house is bought, and the value of the non-financial asset does not depend on the amount of initial payment and loan period.

**(3) Cash flow tables and other parameters**

We make cash flow tables of three households based on the above-mentioned setting. Three parameters of wage income $m_t$, expenditure $C_t$, non-financial asset $W_{2,t}$ can be calculated by using the cash flow tables. The consumption expenditure ($C_t$) is the sum of the expenditures for life ($C_{1,t}$) and non-financial asset ($C_{2,t}$). The consumption expenditure for life consists of the living expense, educational cost, and rent or loan payment for a house. Due to limitations of space, cash flow tables are omitted. Other parameters are the same in Table 2 except $m_t$, $C_t$, and $W_{2,t}$ set in Table 2. The expected terminal financial asset is set to $W_E = 70$ million yen.

## 6.2 Analysis

We solve the problem and obtain the optimal strategies for three households. We show three graphs: Figure 10 for Household A, Figure 11 for Household B, and Figure 12 for Household C.

**(1) Household A**

As shown in Figure 10, the optimal life insurance is about 87 million yen, the optimal fire insurance is 10 million yen, and the optimal investment ratio of risky asset is about 15% at time 0. The optimal strategy over time shows that the optimal fire insurance jumps up and it rises about 30 million yen, to about 40 million yen when the house is bought (the householder is 40 years old), and it keeps afterwards. This reason is that the household additionally purchases the same amount of the fire insurance as the house price and the optimal fire insurance money is equal to the non-financial asset value at each time as described in Section 5. Our practical feeling coincides with the result. The investment ratio of risky asset is almost constant, however it also jumps up when the house is purchased, and it becomes volatile depending on the event. For example, the investment ratio of risky asset becomes high when the initial payment is done for the house and the educational expense is temporally expensive due to the enrollment fee for children. This reason is that we pay these types of temporal expenditure with risk-free asset, and the ratio of risky asset turns out relatively high among the holding assets.

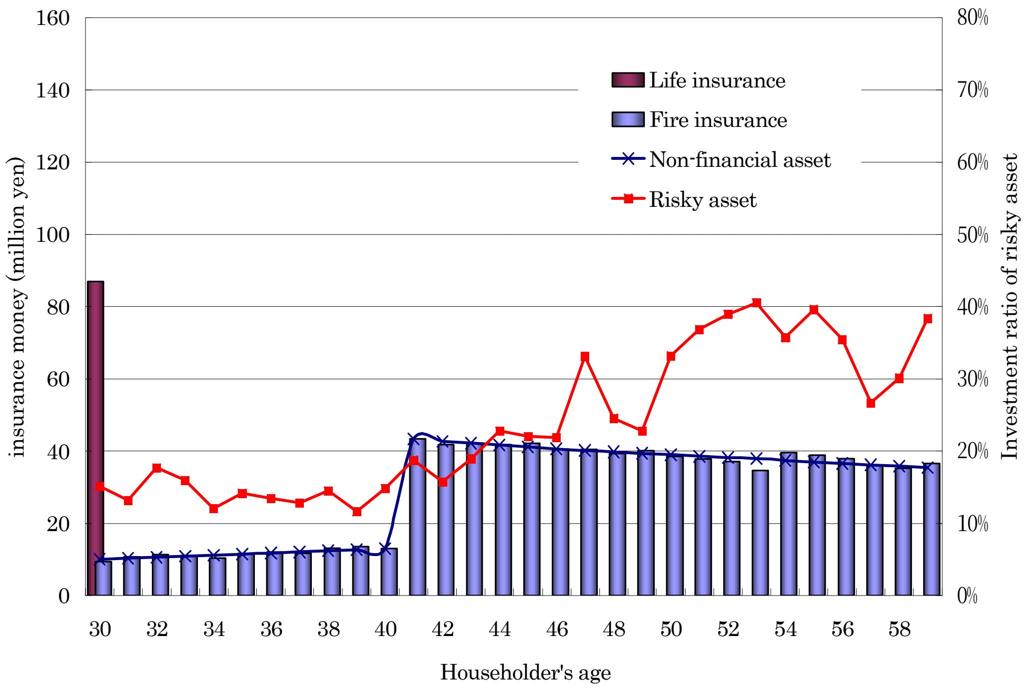


Figure 10: Optimal strategy for Household A

**(2) Household B**

As shown in Figure 11, the optimal life insurance money is about 140 million yen, the optimal fire insurance money is 10 million yen, and the optimal investment ratio of risky asset is about 22% at time 0. The optimal life insurance of Household B is 1.6 times as expensive as that of Household A, because Household A and B have the different wage incomes and consumption expenditures. Life insurance is used to hedge risk associated with the future wage income, and therefore the optimal life insurance of Household B which has the higher cumulative wage income is higher than that of Household A. Moreover, Household B has more children with expensive educational costs, and the higher house price than Household A. Household B needs to buy more expensive life insurance than Household A in order to spend the predetermined money on the children and the house even if the householder dies.

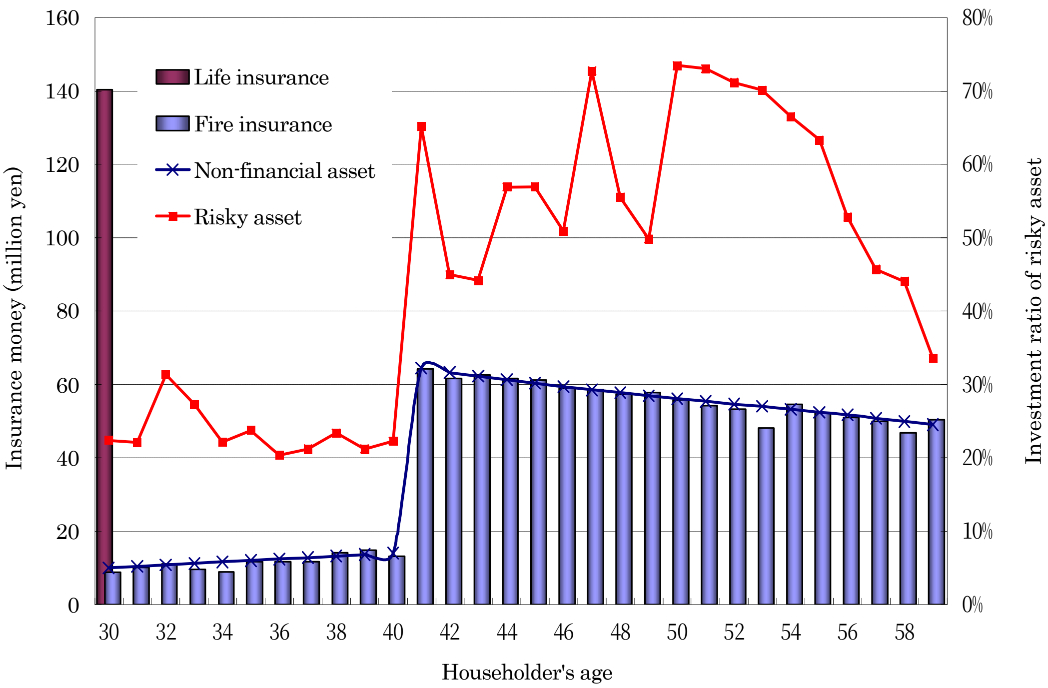


Figure 11: Optimal strategy for Household B

The optimal fire insurance and the optimal investment ratio of risky asset over time are different from those of Household A, but the overall characteristics is similar to Household A. We need to purchase the fire insurance which is equal to the non-financial asset value at the current time, and our optimal strategy is to hold constant ratio of risky asset until the house is purchased. After the house is bought, investment ratio of risky asset jumps up due to the decrease of risk-free asset caused by the temporally expensive expenditure as well as the case of Household A. The reason is that the initial payment of 20 million yen makes up large percentage among the current holding asset. We have to hold the high investment ratio in the range from 50% to 70% until the retirement in order to achieve the expected terminal financial asset of 70 million yen.

**(3) Household C**

As shown in Figure 12, the optimal life insurance is about 300 million yen, the optimal fire insurance is 10 million yen, and the optimal investment ratio of risky asset is about 15% at time 0. A series of the optimal fire insurance over time is the same as the others, however the optimal life insurance is much bigger than the others. This reason is that Household C has more cumulative wage income and consumption expenditure than the others.

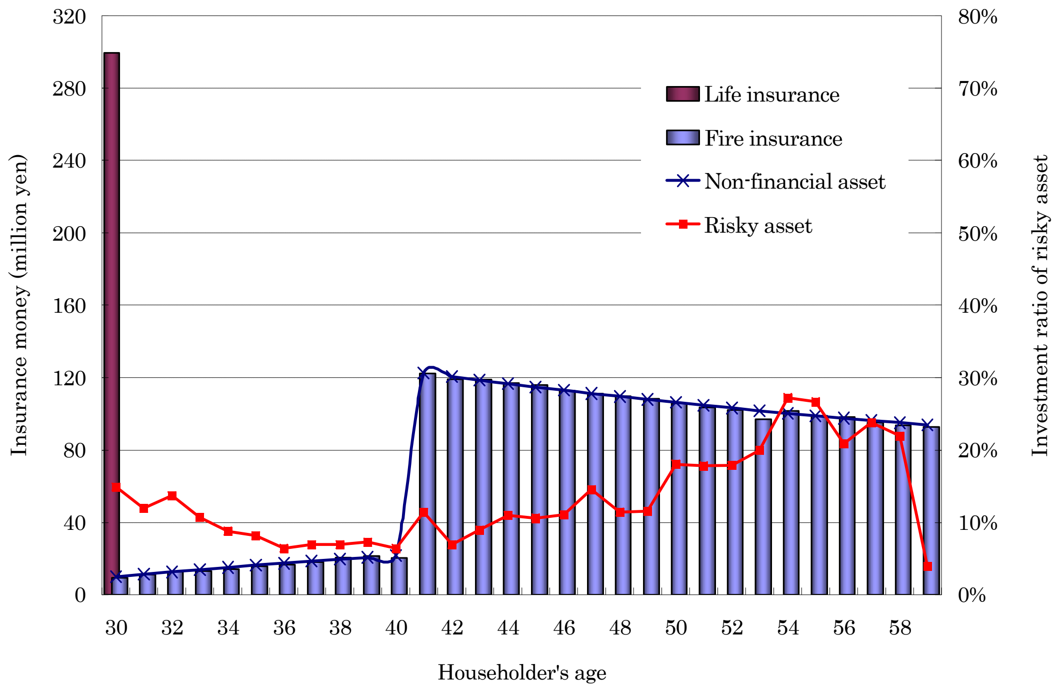


Figure 12: Optimal strategy for Household C

The analysis shows that the optimal insurance strategy and investment strategy depend on the attribution and life cycle of the household. We should give an financial advice and we can tailor the optimal strategy for households.

# 7 Concluding remarks

In this paper, we formulate the ALM optimization model for a household using the multiperiod simulated path approach. The household faces three kinds of risk factors: the death of the householder, the fire of the house, and inflation. We show the life insurance to hedge death risk for a householder, the fire insurance to hedge fire risk, and the risky asset to hedge inflation risk. We describe the multiperiod optimization model to determine the optimal investment and insurance strategy, and we build up assets stably by hedging various risk in conjunction with the life cycle until the householder retires.

Our numerical examples show the practical results as follows;

① The older the householder is, the less the optimal life insurance is, because the life insurance can hedge risk associated with the wage income earned in the future.

② The optimal fire insurance is equal to the maximum loss of non-financial asset because the fire insurance can hedge risk against loss of non-financial asset.

We can obtain an interesting result that inflation risk does not affect the optimal life insurance and the optimal fire insurance because the life insurance and the fire insurance have the ability to hedge risk which cannot be substituted by other financial products. Inflation risk is described by the increase of the expected terminal financial asset in this paper. We illustrate the optimal investment and insurance strategy which reflect the life cycle and the attribution of the household such as wage income earned by the householder, purchase of the house, the number of children. We can show the usefulness of the model with the practical examples.

We examine that the model can be served as the financial consulting tools. It becomes more important to use the tools for giving a financial advice. We expect this type of the model will be used widely in practice.

## Reference


Bodie, Z. and D.B. Crane(1997), "Personal Investing:Advice, Theory, and Evidence", *Financial Analyst Journal*, November/December, 13–23.

Bodie, Z., R.C. Merton and W. Samuelson (1992), "Labor Supply Flexibility and Portfolio Choice in a Life-Cycle Model", *Journal of Economic Dynamics and Control*, **16**, 427–449.

Hibiki, N. (2000), "Multi-Period Stochastic Programming Models for Dynamic Asset Allocation", *Proceedings of the 31st ISCIE International Symposium on Stochastic Systems Theory and Its Applications*, pp.37–42.

Hibiki, N. (2001a), Financial engineering and optimization, Asakura Shoten (in Japanese).

Hibiki, N. (2001b), "Multi-Period Stochastic Programming Models Using Simulated Paths for Strategic Asset Allocation", *Journal of Operations Research Society of Japan*, **44**, 169–193 (in Japanese).

Hibiki, N. (2001c), "A Hybrid Simulation/Tree Multi-Period Stochastic Programming Model for Optimal Asset Allocation", Takahashi, H. (eds.), *The Japanese Association of Financial Econometrics and Engineering, JAFEE Journal [2001]*, pp. 89–119 (in Japanese).

Hibiki, N. (2003), "A Hybrid Simulation/Tree Stochastic Optimisation Model for Dynamic Asset Allocation", Scherer, B. (eds.), *Chapter 14 in Asset and Liability Management Tools: A Handbook for Best Practice, Risk Books*, pp. 269–294.

Hibiki, N. (2005), Multi-period Stochastic Optimization Models for Dynamic Asset Allocation, Journal of Banking and Finance, accepted.

Merton, R.C. (1969), "Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case", *Review of Economics and Statistics*, **51**, 247–257.

Merton, R.C. (1971), "Optimum Consumption and Portfolio Rules in a Continuous-Time Model", *Journal of Economic Theory*, **3**, 373–413.

Merton, R.C. (1992), *Continuous-Time Finance*, Blackwell.

Ministry of Education, Culture, Sports, Science and Technoloty (2003), "The survey of household expenditure on education per student(2002)", http://www.mext.go.jp/b_menu/toukei/001/006/03121101.htm .

Ministry of Education, Culture, Sports, Science and Technology (2003), "The survey of student life(2002)", http://www.mext.go.jp/b_menu/houdou/16/04/04040702.htm .

Ministry of Health, Labour and Welfare (2003), "Wage income of the household over time based on the Census of wage (2003) ",
http://wwwdbtk.mhlw.go.jp/toukei/kouhyo/indexk-roudou.html .

Rockafellar R,T and S.Uryasev (2000), "Optimization of conditional value-at-risk", *Journal of Risk*, **2**, 21–41.

Samuelson, P.A. (1969), "Lifetime Portfolio Selection by Dynamic Stochastic Programming", *Review of Economics and Statistics*, **51**, 239–246.

Statistic Bureau, Ministry of Internal Affairs and Communications, "The national survey of family income and expenditure(1999) ",
http://www.stat.go.jp/data/zensho/1999/021index.htm .

The Central Council for Financial Services Information (2003), "Public Opinion Survey on Household Financial Assets and Liabilities(2003)",
http://www.saveinfo.or.jp/kinyu/yoron/2003/03yoron.html .

The Institute of Acturaries of Japan (1996), "Life insurance standard life table 1996".

Yoshida, Y., Yamada, Y. and Hibiki, N. (2002), "Multi-Period optimization model for household asset allocation problem", Department of Administration Engineering, Keio University, Technical report, No.02-003.