Multi-Period Investment Policy for Corporate Pension Fund with Sponsoring Company

Muneki Kawaguchi *and Norio Hibiki †

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Abstract

We propose the optimization model to obtain multi-period corporate pension investment strategy that reflects both short-term and long-term views, and we analyze the impact of the optimal strategy for the sponsoring company. We generate asset returns based on the economic phases in the model, which involves both those two types of views. We investigate the sensitivity to investment strategies determined by the conditions of pension fund and sponsoring company.

1 Introduction

It is a very important issue how to invest pension assets on corporate pension management. Pension managers have to solve various issues about investment policy and strategy on their pension assets.

The most important issue is to enhance the certainty of future pension benefits on pension management. Participants are involved in the pension systems for a long time, and it is more than 50 years from the entry until the payments of pension benefits are completed. Therefore, experience assumptions used for actuarial valuation such as discount rate and guaranteed interest rate, are determined based on long-term average. Actuarial gains and losses are adjusted over the long term(5-20 years). Pension systems are designed from a long-term perspective.

The deferred recognition of gains and losses will not be allowed in the near future because of the review of accounting standard for retirement benefits in Japan. This means that pension funds are also required to manage their assets from a short-term standpoint in addition to the long-term management. It will be important features to manage the pension system from both standpoints.

The relationship between investment strategies and investment horizon has been studied. Thorley(1995) said Mean-variance optimizers are not indifferent to time horizon, and investment horizon indifference is only a special case within expected utility theory. Also Basak and Chabakauri(2010) provided a fully analytical simple characterization of the dynamically optimal mean-variance portfolios within a general incomplete-market economy. In short, this indicates it is good to control the asset allocation dynamically depending on the situation at that time. On the other hand, Levy and Duchin(2004) tested the goodness of fit of 11 theoretical distributions for investment horizons ranging from one day to four years. They got the result that the normal distribution performs poorly, and never

*Mitsubishi UFJ Trust Investment Technology Institute Co., Ltd.(MTEC), E-mail: kawaguchi@mtec-institute.co.jp

†Faculty of Science and Technology, Keio University, E-mail: hibiki@ae.keio.ac.jp

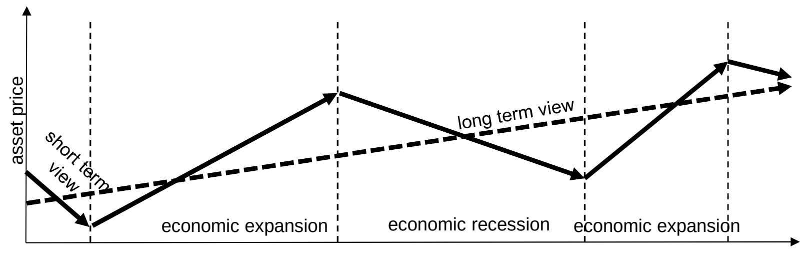


Figure 1: two types of view : short term view and long term view

provides the best fit for any time interval. Like these, it is important that we consider the time horizon to build the investment strategy.

In the business, short-term investment and long-term investment are different in terms of the investment policy such as Figure 1. In the case of long-term investment, the investment policy is based on the average economic condition including good condition and bad condition. On the other hand, in the case of short term investment, it is based on the economic condition at the time.

In this paper, we proposed the model that includes both short-term view and long-term view to build the investment strategy. The model indicates how we should invest in short-term based on both short-term and long-term investment policies.

For example, we think the situation to have to manage the pension fund in five years. If we have the view that the economy turns worse in next year, it is obvious that low risk investments are better in first year. But we cannot achieve the target return without high risk investment after the first year if we reduce the investment risk more than required in the first year. We can invest efficiently by building the dynamic investment strategy depending on both the short-term and long-term view.

Our model gives the optimal pair of short-term strategy and long-term strategy. The difference between the short-term and the long-term strategy indicates the sensitivity of the investment strategy to the changes of economic condition. We analyze what kinds of pension funds should adopt a more sensitive strategy and how much we can improve the efficiency of investment strategy.

We consider the characteristics of the sponsoring company when we decide the investment strategy. On economic recession phase, both the pension finance and the business of the sponsoring company may get worse. In this case, the pension funds have to reduce their investment risk to decrease the bad influence to their sponsoring companies. Kawaguchi and Hibiki(2014) showed that the static investment strategies which suit to the pension funds are different each other by considering the characteristics of sponsoring company. Based on such idea, we analyze how much the sensitivities of the investment strategies are different by the characteristics of the sponsoring companies.

We use the regime switching model which is widely used in the field of financial engineering to express the difference of asset returns under the economic condition [1].

And we use the multi-period optimization model proposed in Hibiki(2006) to obtain the optimal dynamic investment strategy from sample paths of asset returns.

There are the two contributions of this paper. The first is to propose the method that the short-term and long-term views are reflected to the asset return model. Though there are many research on the phases of asset return, as far as we know, there have not existed the researches proposing how to reflect the views. The second is to discuss

[1] Ishijima(2005) is detailed about regime switching model.

about the sensitivity of investment strategies to economic condition by obtaining the dynamic investment strategy quantitatively. We analyze how the differences among pension funds and the differences among sponsoring companies cause the differences of dynamic investment strategy.

This paper is organized as follows. Section 2 presents the model of asset returns and the model to optimize the investment strategies. The model parameters are estimated from market dates in section 3. In section 4, we obtain the optimal investment strategies by our model, and discuss how the characteristics of sponsoring company affect the optimal investment strategies. Section 5 is the conclusion.

2 Model

We use the regime switching model to generate the sample paths of asset returns with some phases, and the multi-period optimization model to obtain the optimal investment strategies. Also, we use the model of pension liabilities.

2.1 Asset Return

We generate asset returns by the regime switching model which can express the distributions of asset returns and the correlations between these returns in response to changes in economic situations. We assume that the statistics of asset returns are varied with stochastic process, called regime, which is not observable directly. We use the regime switching model with two regimes in this paper [2].

We explain the discrete time model. The process Y_t is assigned to either the state of economic expansion ($Y_t = 1$) or economic recession ($Y_t = 2$). The transition probability q_{kl} from state k to state l is expressed by $q_{kl} = \mathbf{P}[Y_t = l | Y_{t-1} = k] (k, l = 1, 2)$, and we assume it is independent from time.

If the probability of the state k at time t is p_{tk}, the probability of the state l at time $t + 1$ is calculated by

$$p_{(t+1)l} = \mathbf{P}[Y_{t+1} = l] = \sum_{k=1}^{2} \mathbf{P}[Y_{t+1} = l | Y_t = k] \mathbf{P}[Y_t = k] = \sum_{k=1}^{2} q_{kl} p_{tk}. \tag{1}$$

The state probability is converged to a certain probability (called limiting probability) after time passes enough. We express such probability as $p_k^* (k = 1, 2)$ in Equation (2),

$$p_k^* = \sum_{l=1}^{2} q_{kl} p_l^* \quad (k = 1, 2). \tag{2}$$

From Equation (2) and $p_1^* + p_2^* = 1$, we obtain

$$p_1^* = \frac{q_{12}}{q_{12} - q_{11} + 1} \qquad p_2^* = \frac{q_{21}}{q_{21} - q_{22} + 1}. \tag{3}$$

Suppose a vector of random variables $R_t \in \mathbb{R}^M$ denotes asset returns, which follow M-dimensional normal distribution with mean return vector $\mu^k \in \mathbb{R}^M$ and covariance matrix $\Sigma^k \in \mathbb{R}^{M \times M}$ for regime k as in Equation(4),

$$R_t | (Y_t = k) \sim N(\mu^k, \Sigma^k) (k = 1, 2). \tag{4}$$

[2] There is the regime switching model with more than three regimes. We restrict to only two regimes due to the following two issues in the analysis. The one is that the number of data is very small, and the other is that it is difficult to interpret the results derived for more than three regimes.

Given a state probability p_{tk}, the averages and the standard deviations of asset returns can be calculated as

$$\mathbf{E}[R_{tj}] = \sum_{k=1}^{2} \mu_j^k p_{tk}, \tag{5}$$

$$\mathbf{V}[R_{tj}] = \sum_{k=1}^{2} (\sigma_j^k)^2 p_{tk} + (\mu_j^1 - \mu_j^2)^2 p_{t1} p_{t2}. \tag{6}$$

where μ_j^k is the expected rate of return of asset j at regime k, and σ_j^k is the standard deviation. The long-term investment strategy is decided using the long-term view about average returns and the risks of the investment assets. We assume that the statistical parameters of the limiting distribution of asset returns are estimated in accordance with the long-term view. The parameters estimated using historical asset returns are adjusted based on the long-term view under the assumption. The short-term view is reflected into asset allocation by the adjustment of the state probability.

These parameters can be estimated by EM algorithm, which is an iterative method for finding the maximum likelihood estimates of parameters iteratively. It alternates between performing an E(expectation) step and a M(maximization) step. Refer to Ishijima(2005) in detail.

2.2 Multi-period Stochastic Optimization Model

We find the optimal investment strategies using asset returns generated by the regime switching model in the previous subsection. We examine the relationship between the economic phases and investment strategies. If the parameter estimates of distributions of asset returns are determined depending on the economic phase, the investment strategies may be decided, according to the information. But, we suppose we cannot make use the information about economic phases in our model, because it may be difficult for investors to specify the economic phase exactly [3].

We call the investor's view about the current economic phase the "short-term view", and call the investor's view about the estimates of asset returns the "long-term view". Also, we formulate the model so that the short-term investment strategy in the first period can be different from the long-term investment strategy after the second period.

We define the change in the funding ratio as the surplus return in order to evaluate the investment strategies, and the surplus risk is defined as the 95% CVaR of the surplus return in this paper. When we involve the risk of sponsoring company, we define the total risk as the 95% CVaR of the sum of the change in the net assets of sponsoring company plus the pension surplus. The reasons why we use CVaR as the risk measure are (1) we implement multi-period investment strategies, (2) the asset returns do not necessarily follow the multi-dimensional normal distribution. We minimize the surplus risk or the total risk subject to the lower bound constraint for expected surplus return.

The multi-period optimization model proposed in Hibiki(2001) gives the optimal investment strategy under the simulated sample paths generated by Monte Carlo method or other methods. The model has the three types of formulations on which the investment strategies are expressed by investment amounts, investment units and investment ratio, respectively. Hibiki(2006) proposed the model with the investment unit function to describe the three types of formulation.

[3] When we can use the information, we obtain the investment strategies according to the information by the approach proposed in Hibiki(2006).

When investment strategies are expressed by investment ratio on the optimization problem, it is more difficult than other types of formulations to solve the problem because the optimization problem includes the nonlinear and non-convex constraints. Hibiki(2006) has proposed the iterative method to solve the optimization problem as follows.

In the first iteration, we get the optimal strategy of investment amounts, and calculate the pension amounts on sample paths. In the next iteration, we use the pension amount computed in the first iteration to express the investment ratio. Therefore we can formulate the model with linear constraints, and easily solve the problem. We iterate the computing process in the same way, and we can obtain the investment strategy expressed by investment ratio.

We describe the formulation of multi-period optimization problem as follows. The parameter k is the number of iterations. When $k = 1$, the investment strategy is expressed by investment units. When $k \geq 2$, the investment strategy is expressed by investment ratio. $\hat{P}_{it}^{A,k}$ indicates the pension assets on path i at k-th iteration.

- Parameters

k	number of iterations(≥ 1)
N	number of sample paths
M	number of risky assets
p_{j0}	price of risky assets j at time 0 $(j = 1, \cdots, M)$
p_{ijt}	price of risky assets j of path i at time t $(i = 1, \cdots, N; j = 1, \cdots, M; t = 0, \cdots, T)$
P_0^A	pension assets at time 0
P_0^L	pension liabilities at time 0
P_{iT}^L	pension liabilities of path i at time T $(i = 1, \ldots, N)$
P_0^C	net assets of sponsoring company at time 0
P_{iT}^C	net assets of sponsoring company of path i at time T $(i = 1, \ldots, N)$
$\hat{P}_{it}^{A,k}$	pension assets of path i at time t by k-th investment strategy $(i = 1, \ldots, N; t = 1, \ldots, T; k \geq 1)$
r_{it}	interest rate of path i in period t $(i = 1, \ldots, N; t = 1, \ldots, T)$
NCF_t	net cash flow (pension contribution $-$ pension benefit) to pension fund at time t $(t = 1, \cdots, T)$
$\underline{\Delta FR}$	lower limit for expected annual change in the funding ratio

- Decision variables

z_j^s	investment ratio to asset j at time 0(in the first period) $(j = 1, \ldots, M)$
z_j^l	investment ratio to asset j used commonly from time 1(the second period) to time $T-1$(the T-th period) $(j = 1, \ldots, M)$
v_0	cash amounts at time 0
v_{it}	cash amounts of path i at time t $(i = 1, \ldots, N; t = 1, \ldots, T)$
P_{iT}^A	pension assets of path i at time T $(i = 1, \ldots, N)$
VaR_β	VaR of surplus at a β% confidence level
u_i	deviation of the surplus loss of path i below VaR_β at time T $(i = 1, \ldots, N)$

$$\textbf{Minimize} \qquad VaR_\beta + \sum_{i=1}^{N} \frac{u_i}{(1-\beta)N} \tag{7}$$

subject to (8)

$$v_0 + \sum_{j=1}^{M} h(i,j,k,0)p_{j0} = P_0^A \qquad (i=1,\dots,N) \tag{9}$$

$$v_0(1+r_{it}) + \sum_{j=1}^{M} h(i,j,k,0)p_{ij1} + NCF_1 = v_{i1} + \sum_{j=1}^{M} h(i,j,k,1)p_{ij1} \tag{10}$$

$$(i=1,\dots,N) \tag{11}$$

$$v_{i(t-1)}(1+r_{it}) + \sum_{j=1}^{M} h(i,j,k,t-1)p_{ijt} + NCF_t = v_{it} + \sum_{j=1}^{M} h(i,j,k,t)p_{ijt} \tag{12}$$

$$(i=1,\dots,N; t=2,\dots,T-1) \tag{13}$$

$$v_{i(T-1)}(1+r_{it}) + \sum_{j=1}^{M} h(i,j,k,T-1)p_{ijT} + NCF_T = P_{iT}^A \quad (i=1,\dots,N) \tag{14}$$

$$P_{iT}^C - P_0^C + P_{iT}^A - P_0^A - P_{iT}^L + P_0^L + u_i + VaR_\beta \geq 0 \quad (i=1,\dots,N) \tag{15}$$

$$\frac{1}{T}\frac{\sum_{i=1}^{N} P_{iT}^A}{\sum_{i=1}^{N} P_{iT}^L} - \frac{P_0^A}{P_0^L} \leq \underline{\Delta FR} \tag{16}$$

$$0 \leq z_j^s \leq 1 \quad (j=1,\dots,M) \tag{17}$$

$$0 \leq z_j^l \leq 1 \quad (j=1,\dots,M) \tag{18}$$

$$v_0 \geq 0 \tag{19}$$

$$u_i \geq 0 \quad (i=1,\dots,N) \tag{20}$$

The function h in the formulation is called investment unit function,

$$h(i,j,k,t) = \begin{cases} z_j^s & k=1, t=0 \\ \frac{\hat{P}_{it}^{A,k-1}}{p_{ijt}} z_j^s, & k \geq 2, t=0 \\ z_j^l & k=1, t \geq 1 \\ \frac{\hat{P}_{it}^{A,k-1}}{p_{ijt}} z_j^l, & k \geq 2, t \geq 1 \end{cases} \tag{21}$$

The objective function in the formulation is the total risk to be minimized. If we minimize the surplus risk, we set $P_0^C = 0$, $P_{iT}^C = 0 (i=1,\dots,N)$.

We discuss both static and dynamic investment strategies. We impose the constraint of $z_j^s = z_j^l (j=1,\dots,M)$ for the static investment strategy. On the other hand, we do not impose the constraints with respect to the decision variables of z_j^s and z_j^l for the dynamic investment strategy.

2.3 Pension Liability

We use the model of pension liability in Kawaguchi and Hibiki(2014). We assume the pension system has entry age a, retirement age b and maximum vesting age c, and the same number of pensioners of each generation. Also we assume a member keeps a unit of the amounts of pension reserve at retirement. The sponsoring company contributes $1/(b-a)$ units per member annually, and pays $1/(c-b)$ units per pensioner annually for pension benefit. Under these assumptions, Equation (22) shows pension benefits, CF_t, that the sponsoring company must pay at time t.

$$CF_t = \begin{cases} \frac{c-b+1-t}{c-b} + \frac{1}{(c-b)(b-a)} \left[(t-1)(b-a+1-t) + \frac{1}{2}(t-1)(t-2)\right] & t = 1, \dots, c-b \\ \frac{1}{b-a} \left[(b-a+1-t) + \frac{1}{2}(c-b-1)\right] & t = c-b+1, \dots, b-a \\ \frac{1}{(c-b)(b-a)} \left[(c-a-t) + \frac{1}{2}(c-a-t)(c-a-1-t)\right] & t = b-a+1, \dots, c-a-1 \end{cases} \quad (22)$$

The pension liabilities are the present values of the pension benefits,

$$\sum_{t=1}^{c-a-1} CF_t DF_t, \quad (23)$$

where DF_t is the discount factor at time t.

Given the target investment return of pension assets y, the pension premium per member C is calculated by the following equation.

$$\sum_{t=1}^{b-a} C(1+y)^{b-a-t} = 1. \quad (24)$$

The pension premium to the whole pension fund is the number of generations of pension members $(b-a)$ times C, that is, $(b-a)C$. On the other hand, the whole pension contribution is just 1, which is pension contribution per pensioner $1/(c-b)$ times the number of generations of pensioners $(c-b)$.

3 Parameter Estimation

We explain how to estimate the parameters of our model in this section. In subsection 3.1 and 3.2, we explain how to estimate the parameters of the return distribution with respect to risky assets and business of sponsoring company. The regimes are estimated as economic phases using asset returns. Essentially, it should be estimated involving the returns of the sponsoring company's business at the same time. However, the returns of business are observed quarterly or semiannually, and it is difficult statistically to estimate the economic phases.

The parameters are estimated from historical data in subsection 3.1 and 3.2, but we need to modify them based on the future views to decide asset allocation. In subsection 3.3 and 3.4, we explain how to incorporate short- and long-term views in the parameters estimated from historical data.

3.1 Asset returns on economic phase

We estimate the economic regimes by the regime switching model using the historical returns for five assets; domestic stock, domestic bond, foreign stock, foreign bond and cash in this paper [4].

[4]We use the following monthly indexes from March 1998 to March 2013 (300 months) as the returns of assets. domestic stock : TOPIX(including dividend), domestic bond : NOMURA-BPI, foreign stock : MSCI-KOKUSAI(Japanese yen base), foreign bond : Citi WGBI(Japanese yen base), cash : secured call loan rate.

On the other hand, the pension liability is the sum of the present values of the discounted future pension benefits by the Japanese treasury yield. The future pension benefits are calculated by the pension liability model. We calculated the pension liability using histrical monthly yield curve to estimate the monthly change of pension liability. We assume the pension fund receives contributions from members between 20 and 60 years old and pay benefits to pensioners between 60 and 80 years old, and it has the same number of pensioners of each generation.

We use the regime switching model, and find the maximum likelihood estimates of the parameters of asset returns are estimated by EM algorithm. The first 1,000 samples are thrown away to eliminate the initial value dependence on the estimation because initial samples usually may not follow a desired distribution. Figure 2 shows the estimates of the parameters of each asset [5]. The solid lines in Figure 2 indicate the monthly asset returns, and the shaded areas show the state probabilities on one of the two phases. The non-shaded areas show the state probabilities on alternative phase.

The economic phases of domestic stock and foreign stock have switched every 1 to 5 years. It is interesting that the regimes of these assets are switched almost coincidentally after 2003. This coincidence indicates the relationship between stock markets became stronger. Also the phases of domestic bond switched exactly at between 1999 and 2000. Foreign bond has the one phase only during 1991-1993 and 2005-2008 and the alternate phase during remaining periods.

We define the common process of phases among these assets for the following analysis because it is difficult to derive the optimal allocation under the condition that each asset has the different process of phases, respectively. We employ the process of phases of domestic stock. The reasons are that the domestic stock has the highest risk among assets and the similar process of phases to the foreign stock.

Also we define the months with more than 50% state probability as the period with economic expansion phase, and the other months as the periods with economic recession phase in Figure 2(a). We can show the difference of the average annual return of domestic stock between two phases, which is 19.35% in expansion phase, and −22.43% in recession phase.

Table 1 shows average returns and volatilities of assets, and correlations between assets in each economic regime, respectively. The average returns of domestic and foreign stocks are larger in expansion phase than in recession phase, while those of pension liability and domestic bonds are less in expansion phase. This result indicates that interest rates rise on expansion phase.

The volatilities of all assets are higher in recession phase, and also they tend to become larger when their returns are lower. The correlations are not different between in expansion and in recession. Generally, it is said that the correlations go up when the economy goes down. The structure is implied in the difference of the volatilities between phases in our model.

Next, the annual transition probabilities are indicated in Table 2. The probability that the following year is in expansion phase is 74.0% when current year is in expansion phase. When current year is in recession phase, such probability is 69.6%. Hence the difference of these probabilities is only about 5%, and the phase in current year does not almost affect the phase in the following year.

Further the state probabilities given by Equation (3) are 72.8% for the expansion phase, and 27.2% for the recession phase.

[5] We can estimate the parameters of four assets simultaneously. However, we paid attention to the instability of the estimates caused by the short data periods which are only 300 months.

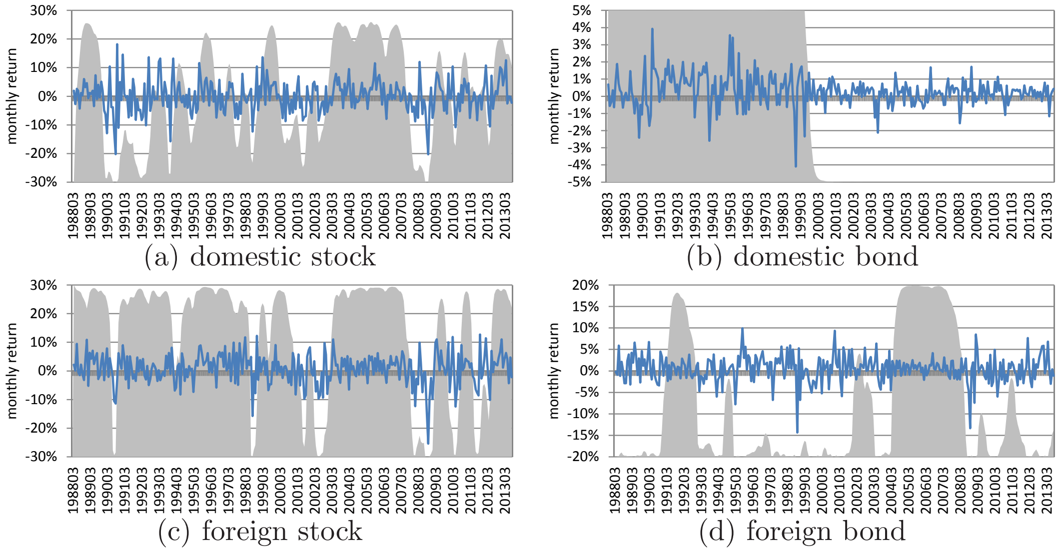


Figure 2: Regimes estimated from asset returns

Table 1: Statistics of asset returns for the entire period and two regimes(per annual)

(a) Entire period

	average return	standard deviation	correlation					
			pension liability	domestic stock	domestic bond	foreign stock	foreign bond	cash
pension liability	2.13%	8.40%	1.00	−0.07	0.92	−0.03	0.07	0.01
domestic stock	0.45%	19.56%	−0.07	1.00	−0.07	0.49	0.15	−0.07
domestic bond	3.53%	3.09%	0.92	−0.07	1.00	−0.01	0.06	0.15
foreign stock	9.92%	18.54%	−0.03	0.49	−0.01	1.00	0.60	0.00
foreign bond	6.45%	10.72%	0.07	0.15	0.06	0.60	1.00	0.00
cash	1.45%	0.66%	0.01	−0.07	0.15	0.00	0.00	1.00

(b) Economic expansion regime

	average return	standard deviation	correlation					
			pension liability	domestic stock	domestic bond	foreign stock	foreign bond	cash
pension liability	−0.53%	7.20%	1.00	−0.01	0.94	0.01	0.12	−0.05
domestic stock	19.35%	13.82%	−0.01	1.00	0.03	0.49	0.16	0.06
domestic bond	2.07%	2.52%	0.94	0.03	1.00	0.04	0.08	0.13
foreign stock	22.09%	14.67%	0.01	0.49	0.04	1.00	0.60	0.09
foreign bond	6.77%	9.41%	0.12	0.16	0.08	0.60	1.00	−0.02
cash	0.81%	0.43%	−0.05	0.06	0.13	0.09	−0.02	1.00

(c) Economic recession regime

	average return	standard deviation	correlation					
			pension liability	domestic stock	domestic bond	foreign stock	foreign bond	cash
pension liability	5.33%	9.56%	1.00	−0.09	0.89	−0.03	−0.02	0.02
domestic stock	−22.43%	23.06%	−0.09	1.00	−0.12	0.39	0.15	−0.05
domestic bond	5.30%	3.59%	0.89	−0.12	1.00	−0.01	0.03	0.09
foreign stock	−4.81%	21.54%	−0.03	0.39	−0.01	1.00	0.63	0.00
foreign bond	6.05%	12.09%	−0.02	0.15	0.03	0.63	1.00	0.05
cash	2.22%	0.81%	0.02	−0.05	0.09	0.00	0.05	1.00

Table 2: Annual transition probabilities between economic phases

		following year	
		expansion	recession
current year	expansion	74.0%	26.0%
	recession	69.6%	30.4%

Table 3: Statistics of business returns

(a) Averages and volatilities on economic phases

		high−tech	cyclical	domestic demand	defensive	real estate
entire period	average return	3.98%	5.16%	5.51%	6.64%	3.24%
	standard deviation	3.75%	3.69%	2.55%	1.77%	4.14%
expansion	average return	5.24%	7.02%	6.48%	7.64%	4.08%
	standard deviation	2.69%	2.98%	1.80%	1.56%	2.95%
recession	average return	2.82%	3.45%	4.61%	5.72%	2.48%
	standard deviation	4.06%	3.28%	2.71%	1.34%	4.73%
expansion −recession	average return	2.42%	3.58%	1.87%	1.92%	1.60%
	standard deviation	−1.37%	−0.30%	−0.91%	0.22%	−1.78%

(b) Correlation between asset returns and business returns

	pension liability	domestic stock	domestic bond	foreign stock	foreign bond	cash
high−tech	−0.13	0.13	−0.07	0.36	0.37	0.30
cyclical	−0.29	0.06	−0.28	0.16	0.21	0.25
domestic demand	−0.24	0.13	−0.14	0.20	0.18	0.30
defensive	−0.23	0.23	−0.45	−0.11	−0.11	−0.30
real estate	−0.02	−0.01	0.13	−0.02	−0.07	0.61

3.2 Business return of sponsoring company

The business return is defined as return on equity(ROE) as well as Kawaguchi and Hibiki(2014). Specifically, the weighted average ROE is employed. The 671 listed companies in twenty-eight industry sectors except financial sector and electric and gas sector are analyzed to avoid the influence of the Great East Japan Earthquake. Data period is from March 1989 to March 2013. The twenty-eight sectors in Tokyo Stock Exchange are classified into five groups; high-tech, cyclical, domestic demands, defensive, real estate [6].

Next, we estimate the business returns on each economic phase. However, the economic phase is estimated using monthly asset returns, and therefore annual accounting data cannot be assigned to the monthly estimated phases. Then we define the fiscal year which contains more than six economic expansion months as the economic expansion year to estimate the economic phase on an annual basis. The economic recession years are defined as well. According to these definitions, twelve years are in expansion years, thirteen years are in recession years.

Table 3(a) summarizes the averages and volatilities of the business returns by groups. Each group has larger average returns in expansion phase than those in recession phase. Especially, the high-tech and cyclical groups have the larger difference of the averages between economic phases than other groups. All groups except the defensive have smaller volatilities in expansion phase. The high-tech group has the largest difference of volatilities

[6] Each group includes the following industry sectors.

- high-tech(four sectors) : Machinery, Electric Appliances, Transportation Equipments, Precision Instruments
- cyclical(twelve sectors) : Mining, Textiles & Apparels, Pulp & Paper, Chemicals, Oil & Coal Products, Rubber Products, Glass & Ceramics Products, Iron & Steel, Nonferrous Metals, Metal Products, Marine Transportation, Wholesale Trade.
- domestic demands(seven sectors) : Other Products, Electric Power & Gas, Land Transportation, Air Transportation, Warehousing & Harbor Transportation Services, Information & Communication, Retail Trade, Services.
- defensive(three sectors) : Fishery, Agriculture & Forestry, Foods, Pharmaceutical.
- real estates(two sectors) : Construction, Real Estate.

between phases in the five groups.

We do not estimate the correlations on each economic phase, but in common because the number of accounting data is not enough to distinguish the phases, and such estimation is not reliable. Those correlations are shown in Table 3(b). The high-tech and cyclical groups have the higher correlations with foreign assets than other groups. The defensive group has the higher correlation with domestic stock and the negative correlations with foreign assets.

3.3 Long-term view

We obtain the long-term asset allocation based on the long-term view of asset returns. We assume marginal distributions of asset returns in accordance with the long-term view, and we reflect the long-term view into asset allocation by adjusting the parameters estimated from data [7].

The averages and volatilities of asset returns after time passes enough are given by

$$\mu_j^* = \sum_{k=1}^{2} \mu_j^k p_k^*, \tag{25}$$

$$(\sigma_j^*)^2 = \sum_{k=1}^{2} (\sigma_j^k)^2 p_k^* + (\mu_j^1 - \mu_j^2)^2 p_1^* p_2^*, \tag{26}$$

where μ_j^* and σ_j^* are the average return and the volatility of asset j with long-term view, respectively. These equations are derived from Equations (3), (5) and (6).

Given μ_j^* and σ_j^*, the parameters $\mu_j^k, \sigma_j^k (k = 1, 2)$ need to be determined in Equations (25) and (26). But these four parameters cannot be determined uniquely without the additional constraints because of two constraints. We suppose two constraints that $\mu_j^1 - \mu_j^2$ is the same as the difference of the estimates, and σ_j^1/σ_j^2 is the same as the ratio of the estimates, even reflecting the long-term view [8].

The long-term view and the adjusted parameters are shown in Table 4. The long-term view of asset returns are announced by four Japanese trust banks as a view for long-term investment in the financial year of 2013. The long-term view of business returns is presumed the average and risks estimated from entire period data. We can see that these parameters were adjusted while keeping the difference between the economic phases.

3.4 Short-term view

We suppose the short-term view contains the information about the economic condition. The short-term view is reflected into the asset allocation by adjusting the state probabilities.

Table 5 shows the statistics of asset returns and ROE with short-term view. An upper part of each table in Table 5 shows the statistics of asset returns, and a lower part shows the statistics of ROE of the five industry groups.

We assume seven economic views in accordance with the columns from (I) to (VII). The column (I) is the case of the worst economic view, and the column (VII) is the best case. The expected returns of column (IV) represent those with long-term view. Suppose

[7] There are other ways to reflect the long-term view. For example, it can be refected by adjusting the sample paths directly.

[8] The average returns of cash could not be calculated under the constraint that $\mu_j^1 - \mu_j^2$ is the same as the difference of the estimates because the past level of interest rate is much different from the current level. We gave the alternative constraint that μ_j^1/μ_j^2 is the same as the ratio of the estimates.

Table 4: Long-term view and adjusted parameters by long-term view

	long–term view		short–term view			
			expansion		recession	
	average return	standard deviation	average return	standard deviation	average return	standard deviation
pension liability	1.42%	7.70%	−0.18%	6.60%	5.68%	8.76%
domestic stock	6.55%	21.15%	17.91%	8.28%	−23.87%	13.81%
domestic bond	1.00%	2.68%	0.12%	1.99%	3.35%	2.84%
foreign stock	8.08%	22.73%	15.39%	16.85%	−11.51%	24.74%
foreign bond	2.75%	11.33%	2.95%	10.44%	2.22%	13.40%
cash	0.28%	0.60%	0.19%	0.45%	0.51%	0.84%
high–tech	3.98%	3.75%	4.64%	3.10%	2.22%	4.67%
cyclical	5.16%	3.69%	6.13%	3.24%	2.56%	3.56%
domestic demand	5.51%	2.55%	6.01%	2.08%	4.15%	3.13%
defensive	6.64%	1.77%	7.16%	1.61%	5.24%	1.39%
real estate	3.24%	4.14%	3.68%	3.42%	2.08%	5.48%

that the difference of the expected returns of domestic stock between neighboring columns is 2% to derive the statistics with respect to the all economic views except (IV). We can calculate the state probabilities, according to the expected returns of domestic stock. Consequently, the statistics of asset returns except domestic stock can be calculated using the state probabilities.

4 Analysis

In this section, we analyze the optimal investment strategies computed using the sample paths generated by the regime switching model.

4.1 Setting

The pension fund is managed for five years. We define the short- and long-term investment strategies as the strategies in the first year and between the second and the fifth year, respectively.

Suppose three types of pension funds which have the different levels of funding ratios, respectively. The first pension fund is under the lack of funding ratio, and its funding ratio is 66.78%. It is the average funding ratio of 1,408 companies listed in the first section of Tokyo Stock Exchange, and selected by the conditions that they do not included in the four financial sectors, and their pension benefits are more than one billion yen in March 2013 [9]. The second and third pension funds have 20% surplus(120% funding ratio) and just 100% funding ratio, respectively.

Next we explain the parameters with respect to pension funds. These parameters are measured in increments of pension reserves per member at retirement. We need ratios of pension assets, pension liabilities, pension benefits and net assets of sponsoring company for our analysis.

The pension liabilities derived by Equation (23) are 24.1539 units which are the same values among three types of pension funds. The discount factors are calculated using Japanese treasury yield at the end of March 2013. We use 24.1539 units as the pension liabilities P_0^L at time 0. The pension assets are calculated by multiplying pension liabilities and funding ratios. The average ratio of pension liabilities to net assets of sponsoring company is 32.8% in March 2013. The company's net assets P_0^C are 24.1539 ÷

[9] We notice that the funding ratios of pension funds are higher than their actual ratios because pension liabilities include a part of retirement allowance on the Japanese accounting standards.

Table 5: Adjusted parameters by short-term and long-term view

(a) Expected return

	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)
pension liability	2.26%	1.98%	1.70%	1.42%	1.14%	0.86%	0.58%
domestic stock	0.55%	2.55%	4.55%	6.55%	8.55%	10.55%	12.55%
domestic bond	1.46%	1.31%	1.15%	1.00%	0.85%	0.69%	0.54%
foreign stock	4.21%	5.50%	6.79%	8.07%	9.36%	10.65%	11.94%
foreign bond	2.65%	2.68%	2.72%	2.75%	2.78%	2.82%	2.85%
cash	0.32%	0.31%	0.29%	0.28%	0.26%	0.24%	0.23%
high-tech	3.63%	3.75%	3.87%	3.98%	4.10%	4.21%	4.33%
cyclical	4.65%	4.82%	4.99%	5.16%	5.33%	5.50%	5.68%
domestic demand	5.24%	5.33%	5.42%	5.51%	5.60%	5.69%	5.77%
defensive	6.36%	6.46%	6.55%	6.64%	6.73%	6.82%	6.92%
real estate	3.01%	3.09%	3.17%	3.24%	3.32%	3.40%	3.48%

(b) Standard deviation

	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)
pension liability	8.10%	7.98%	7.85%	7.70%	7.54%	7.37%	7.18%
domestic stock	23.31%	22.79%	22.07%	21.15%	19.98%	18.53%	16.72%
domestic bond	2.87%	2.81%	2.75%	2.68%	2.59%	2.49%	2.38%
foreign stock	24.41%	23.93%	23.37%	22.73%	21.98%	21.14%	20.17%
foreign bond	11.77%	11.62%	11.47%	11.33%	11.17%	11.02%	10.86%
cash	0.66%	0.64%	0.62%	0.60%	0.58%	0.55%	0.53%
high-tech	4.01%	3.93%	3.85%	3.75%	3.66%	3.55%	3.44%
cyclical	3.81%	3.78%	3.74%	3.69%	3.63%	3.57%	3.49%
domestic demand	2.73%	2.67%	2.61%	2.55%	2.48%	2.40%	2.32%
defensive	1.79%	1.79%	1.78%	1.77%	1.76%	1.73%	1.71%
real estate	4.46%	4.36%	4.25%	4.14%	4.03%	3.91%	3.78%

(c) State probability

	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)
expansion	58.4%	63.2%	68.0%	72.8%	77.6%	82.4%	87.2%
recession	41.6%	36.8%	32.0%	27.2%	22.4%	17.6%	12.8%

32.8%=73.6399.

The net cash flows of pension fund are given as follows. The expected yield of investment is given 2.41% which is the average value of listed companies in March 2013. The pension benefits to entire generations of pensioners are estimated as 0.6135 by Equation (24). We assume these cash flows are not time-dependent. The net cash flows NCF_t are equal to -0.3865 ($= 0.6135 - 1$). We analyze the optimal investment strategies using above settings.

4.2 Base case

Figure 3 shows optimal strategies and surplus risks when the funding ratio is 100% and the lower limit of expected surplus return is 50bps. The horizontal axis indicates the short-term view, and (I)-(VII) correspond to the views in Table 5. The case (I) has the highest state probability of economic recession, while the case (VII) has the lowest state probability. We obtain the dynamic and static strategies under these settings.

We solve the optimization problems with 5,000 sample paths ten times, and show the average surplus risks in Figure 3(a). The surplus risk is the highest in the case (I) of the worst short-term view. The surplus risk becomes lower as the view improves because of the lower volatilities of asset returns. We can reduce 22bps of surplus risk at a maximum by taking dynamic strategy.

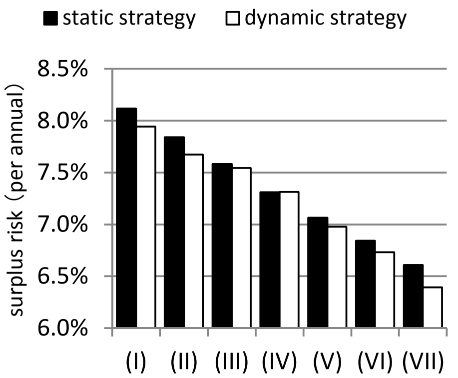
Figure 3(b) shows the asset allocations for static strategy. The lower limit of surplus return tends to cause that the foreign stock has the largest weight in the case (I). The investors allocate the larger weights to the risky assets in order to get the higher returns even the bad economy.

Figure 3(c) and (d) indicate the dynamic allocations in the first year and between the second and the fifth year, respectively. In the case(I), the investors reduce risk by decreasing the weight of foreign stock in the first year due to the bad economy. The investors need to take more risky investment between the second and the fifth years, compared with static strategy to recover the return in the first year. In the case(VII), since the investor takes higher returns in the first year by investing in more domestic and foreign stocks, this strategy can achieve the target return even the low risk investment. By considering both short- and long-term views together, we can get the optimal investment strategies which produce the effects of time diversification.

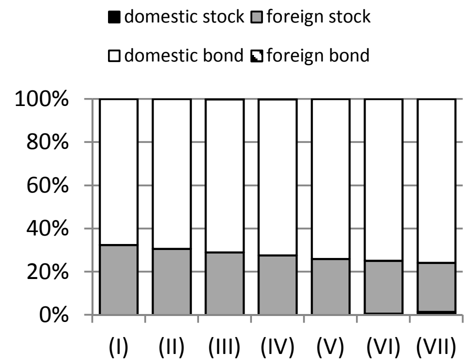
4.3 Pension fund and dynamic strategy

We analyze how the optimal dynamic strategy is affected by funding ratio and lower limit of surplus return. If the asset allocation is affected more by short-term view, the investment strategy is more sensitive to the economic cycle.

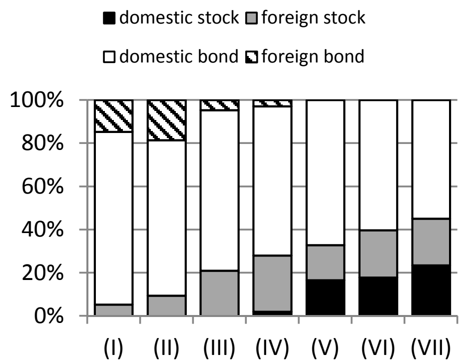
Figure 4 shows the optimal asset allocation in the first year. The results of the 0bps and 50bps of lower limit of surplus returns are shown in Figure 4(a) and (b), respectively. The horizontal axis indicates the cases of the combinations of three kinds of funding ratios of pension fund, two types of strategies(dynamic or static) and three kinds of short-term views. As described before, the case (I) is the worst short-term view with respect to the economic cycle, the case (VII) is the best short-term view, and the case (IV) is that in which the short-term view is the same as the long-term view. Therefore the static asset allocation should coincide with the dynamic asset allocation in the first year in the case (IV). The dynamic strategy in the case (I) (case (VII)) should be riskier (safer) in the first year than that in the case (IV). The similar results are obtained in the every case even if the funding ratios and lower limits of surplus return are different from each other.



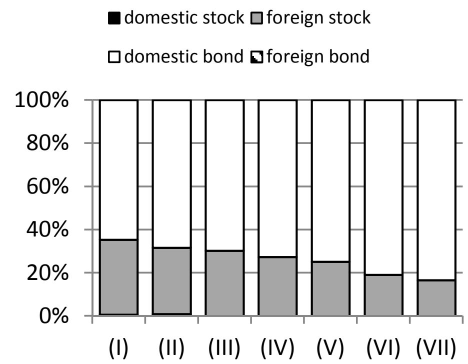
(a) Surplus risk



(b) Asset allocation of static strategy



(c) Asset allocation of dynamic strategy in the first year($t = 0$)



(d) Asset allocation of dynamic strategy between the second and the fifth year($t = 1, \ldots, 4$)

Figure 3: Surplus risk and asset allocation for the fund with 100% funding ratio and 50bps surplus return

Next we focus on the investment strategies due to the difference of funding ratios. When the funding ratio is lower, asset allocation based on the short-term view is sensitive to the funding ratio, and also the optimal investment strategy is sensitive to the economic cycle. For example, it is optimal to invest only in domestic and foreign bonds in the combination of the case (I) and 67% of funding ratio in Figure 4(a). Investors need to invest a half of their assets in domestic and foreign stocks in the case (VII). When the funding ratio is 120%, the optimal allocation consists of only bonds in the case (I), the investors need to invest about 30% in stocks in the case (VII). The lower funding pension fund has to adopt the investment strategy which is more sensitive to the economic cycle in order to get the higher return by investing fewer amounts of assets.

The investors need to increase the investment weights of stocks as thelower limit of surplus return become high. But the differences of asset allocation are almost equal even in the different short-term views. The difference of the weight of stocks between the case (I) and the case (VII) is about 50% in both Figure 4(a) and (b). We interpret this result that the level of target surplus return does not affect the sensitivity of the strategy to the economic cycle.

4.4 Sponsoring company and dynamic strategy

We consider how the optimal dynamic investment strategies are affected by the characteristics of the sponsoring company. Figure 5 shows the asset allocations to minimize the total risk (the sum of business risk and pension investment risk) under 0bps and 50bps of the lower limit of surplus return. The sensitivity of asset allocation to the economic cycle is not affected by the lower limit of surplus return as in Figure 4.

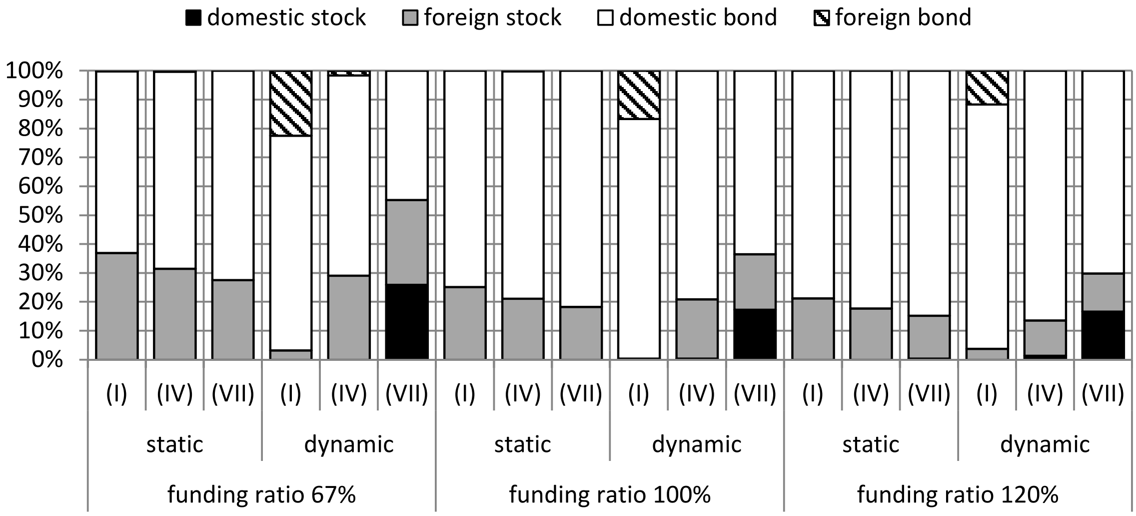
First we focus on the cyclical group and the defensive group. The difference in the standard deviations of cyclical group between column (I) and column (VII) is almost the same as the difference of the defensive group in Table 5. The difference in the average returns of the cyclical group between column (I) and column (VII) is twice as large as the difference of the defensive group. But the optimal asset allocations of two groups almost coincide with each other. This result indicates that the dynamic investment strategies are not sensitive to the difference of expected returns between economic conditions.

Next we focus on the domestic demands group and the defensive group. The difference in the average returns of the domestic demands group between column (I) and column (VII) is almost same as the difference of the defensive group. The difference in the standard deviations of the domestic demands group between column (I) and column (VII) is four times as large as the difference of the defensive group. This is caused by the difference of the standards deviations between economic conditions. Therefore this result indicates that its investment strategy has higher sensitive asset allocation to the economic cycle if the company is more sensitive to the standard deviation of its business.

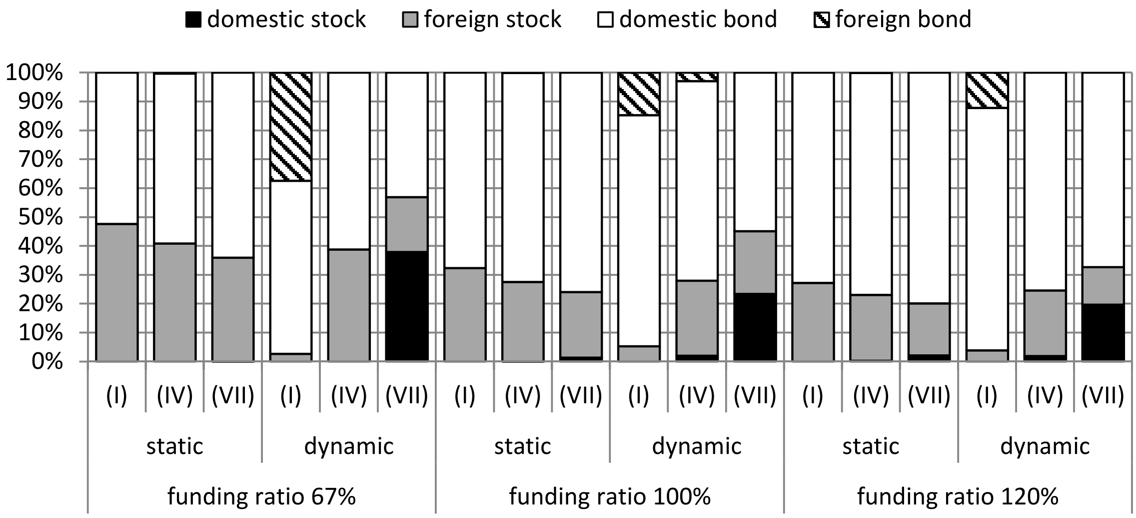
5 Conclusion

In this study, we built the comprehensive model to obtain the investment strategy based on short- and long-term views by the regime switching model. We proposed the approach that the parameters are estimated under the assumption that long-term view is an average view through the economic cycle and short-term view contains the current condition of economic cycle.

Our model shows the risk of the sponsoring company is decreased by dynamic investment strategy. We find that the dynamic asset allocation of sponsoring company should

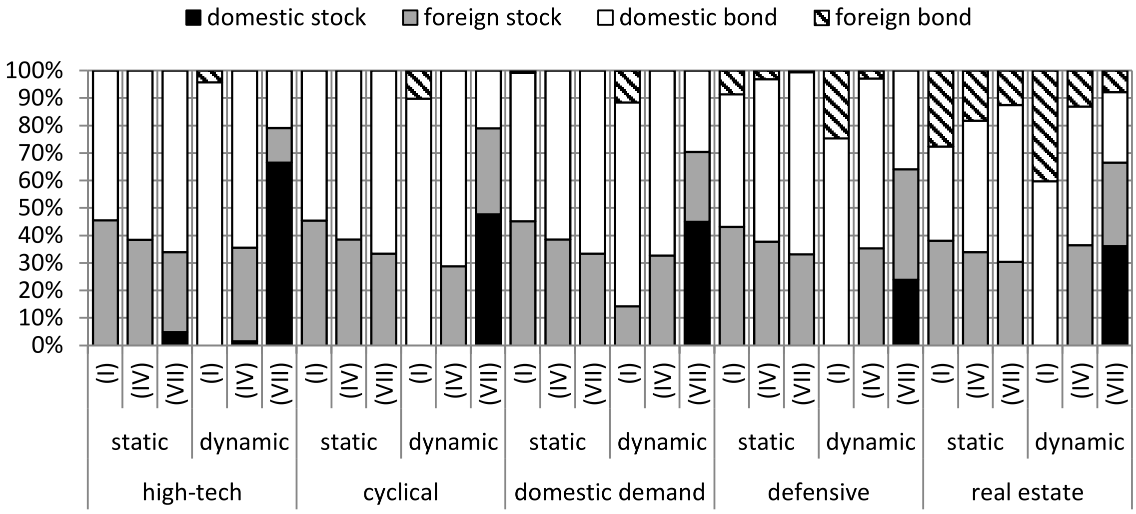


(a) Lower limit of surplus return : 0 bps

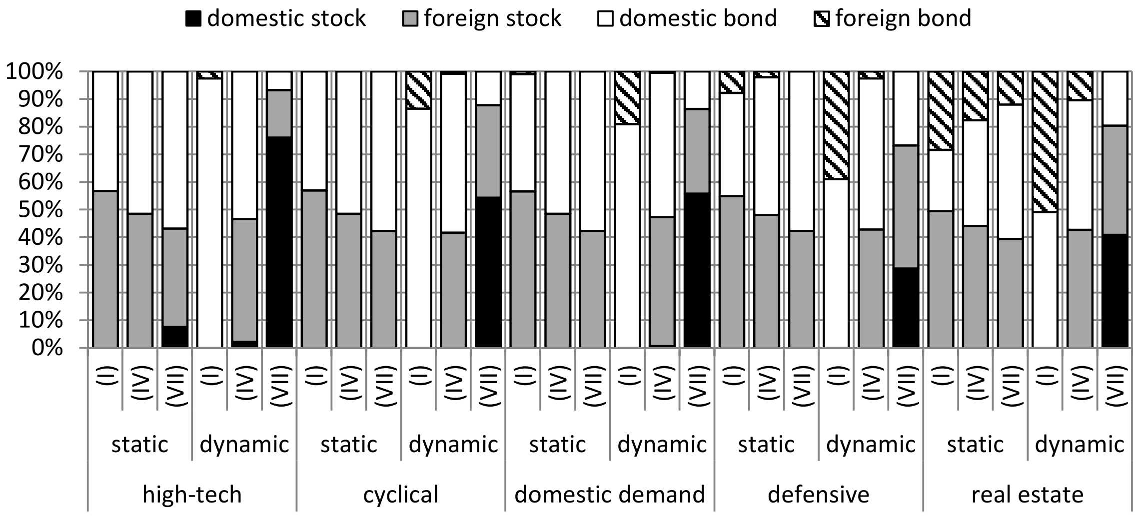


(b) lower limit of surplus return : 50 bps

Figure 4: Relationship between the state of pension fund and asset allocation in the first year



(a) Lower limit for expected surplus return : 0bps



(b) Lower limit for expected surplus return : 50bps

Figure 5: Relationship between the state of sponsoring company and asset allocation in first year

be sensitive to the economic cycle if the sponsoring company has a lower funding ratio and the volatility of business return is sensitive to the economic cycle.

We focus on the effects of the short- and long-term views to the investment strategy. On the other hand, we have to analyze the effects caused by the difference of the structure of pension liabilities in pension management. For example, their benefits become larger than their contributions as the pension fund matures. It is interesting to analyze how asset allocation can be computed in the situation.

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