# Simulation Analysis for Evaluating Risk-sharing Pension Plans

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*Abstract* Recently, occupational pensions with new types of risk-sharing functions have been proposed as alternatives of existing pension plans. They are intermediate plans between DB (defined benefit) and DC (defined contribution) plans, and the associated risks are shared between a sponsoring company and participants. In this paper, we evaluate a risk-sharing pension plan using the quantitative approach. At first, we sort out the concept for risk-sharing, and develop a plan design to share risks among the sponsoring company, the active participants and the retirees. More specifically, we propose a risk-sharing design, which involves a mechanism of sharing the deficiencies and the surpluses in accordance with the funding ratios. Here, we incorporate five parameters to control the level of risk sharing. We evaluate the future uncertainties of the contributions for a sponsoring company, the accrued liabilities for participants, and the benefits for retirees. We employ the expected value and conditional value at risk (conditional tail expectation) as the evaluation measures. We evaluate a risk-sharing pension plan with numerical experiments using the Monte Carlo simulation approach, and compare its outcomes with those of three conventional pension plans; DB, DC, and CB (cash balance) plan.We evaluate a risk-sharing pension plan with numerical experiments using the Monte Carlo simulation approach, and compare its outcomes with those of three conventional pension plans; DB, DC, and CB (cash balance) plan. We also conduct the sensitivity analysis of the five parameters to control the level of risk-sharing, and highlight the characteristics. We implement the backtest using the historical data, and compare four plans.

We find the benefits and contribution payments of the risk-sharing plan are at the level intermediate between DB plan and DC plan because of the risk-sharing features. We also conduct the sensitivity analysis of the five parameters above, and highlight the characteristics of the risk-sharing functions through various simulation analyses. We examine the actual effect on four pension plans through the backtest for twenty years in Japan.

## 1. Introduction

The public pension supports the living expenses in retirement, but the role is expected to be gradually diminished in the future in Japan due to the graying population combined with the low fertility rate. Therefore, the occupational pensions become more important to complement the public pension. However, the traditional DB (defined benefit), DC (defined contribution) and CB (cash balance) plans have some weakness for their sustainability or stability of benefits. In the DB plan, a sponsoring company needs to increase the contribution for lack of the plan asset because of the worse investment condition. But it is forced to increase the contribution under the severe condition due to the decline in the corporate performance because the investment return is dependent on the stock market condition linked to the economic environment. This affects their sustainability, and it could lead to the problem of reducing benefits. There are some cases that the decrease in benefits is accepted actually. In the DC plan, a sponsoring company does not need to make additional payments because participants take investment risk, and accept the decrease in the benefit due to the worse investment condition. But we have a problem concerning stability of the pension which supports the living expenses in retirement. Recently, occupational pensions with new types of risk-sharing functions have been proposed as alternatives of existing pension plans. They are intermediate plans between DB and DC plans, and the associated risks are shared between a sponsoring company, active participants, and retirees.

We introduce some risk-sharing pension plans in several countries. In the U.K., the reformed law went into effect in 2015 in order to introduce the defined ambition pension system (DA plan).

The obligation by a sponsoring company in the DA plan is reduced more than the DB plan, but the more guarantee to active participants is achieved than the DC plan. In Canada, the introduction of "target benefit" plan is proposed in 2014, and the benefit and contribution are adjusted, based on the funding deficiency/surplus. In the Netherlands, the FTK(Financieel Toetsingskader, Financial Assessment Framework) was proposed in 2002, and introduced in 2007. After the global financial crisis, the FTK2 was also proposed in 2011. In Japan, Ministry of Health, Labour and Welfare[13] introduced a basic concept, named "risk-sharing DB plan" to provide a flexible benefit design in September, 2015.

Recently, there are some studies concerning risk-sharing pension plans. Turner[15] evaluates a number of hybrid pension plans. He describes in depth as case studies four different hybrids: the hybrid DB plans in the Netherlands, the nonfinancial DC plan in Sweden, cash balance plans in the United States, Canada and Japan, and the Riester plans in Germany. Hoevenaars and Ponds[8] value intergenerational transfers in collective pension plans. The pension fund is rewritten in generational accounts, and the value-based approach is applied to deal with uncertainty. Three types of policy changes are evaluated: pension plan design in funded collective scheme, investment policy and the setting of the contribution rate. The funding ratio, the contribution rate, and the indexation rate are employed to evaluate them. The combination of the hybrid plan, a 50-50 mix and the fair value approach in setting the contribution rate may be an acceptable midway position amongst the alternatives. Kocken[10] examines two kinds of valuation techniques for pension liabilities in risk-sharing pension plans; liability valuation techniques of state and local pension plans in the U.S.A. and those of the collective defined contribution pension plans in the Netherlands. The two case studies show that arbitrage-free valuation is key to sustainable pension plan design. Kortleve[11] describes a new Dutch pension contract generically labeled defined ambition(DA) plans. The primary advantages of the transition from DB to DA are: (1) shocks are gradually absorbed, and pensions become more stable, (2) plans and their strategies for investment and liability management is focused on indexation ambition, and (3) pension contract prevents shifting underfunding forward, which is good for young participants, and benefit adjustments are gradually balanced, which is good for retirees. Hardy[7] reviews target benefit plan in Canada, and evaluates the risks and benefits of the plan through simulations of economic variables. Kamiyama and Tanaka[18] explore the collective defined benefit pension fund in the U.K.

There are different descriptions in risk-sharing plans of how risk is shared by a sponsoring company, active participants, and retirees. The plan design in practice is discussed mainly in a qualitative manner, but there is little research about the plan designs discussed specifically and quantitatively. In this paper, we evaluate a risk-sharing pension plan using the quantitative approach.

At first, we sort out the concept of risk-sharing, and develop a plan design to share risks among a sponsoring company, active participants and retirees. More specifically, we calculate adjusted contributions born by the sponsoring company, adjusted actuarial liability for active participants, and benefits received by retirees. We propose a risk-sharing design, which involves a mechanism of sharing the deficiency and surplus in accordance with the funding ratio.

Here, we incorporate the following five parameters to control the level of risk-sharing;

1) minimum funding ratio which triggers the deficiency-sharing
2) maximum funding ratio which triggers the surplus-sharing
3) fraction of the deficiency/surplus the sponsoring company will bear/receive
4) annual amortization/decumulation ratio of the deficiency/surplus allocated to the sponsoring company
5) fraction of deficiency/surplus the retirees will share with active participants

It is supposed that we invest a plan asset with a given portfolio. We evaluate a risk-sharing pension plan with numerical experiments using the Monte Carlo simulation approach, and compare its outcomes with those of three conventional pension plans; DB, DC, and CB plan. We also conduct the sensitivity analysis of the five parameters to control the level of risk-sharing, and highlight the characteristics. We implement the backtest using the historical data, and compare four plans. The contributions and characteristics of our paper are in what follows.

(1) Proposal of risk-sharing plan and quantitative evaluation
We formulate the simulation model with parameters of risk-sharing. We run a long-term simulation over a hundred years. For the evaluation, we use the outcomes of the latter sixty years during which the distribution of the plans' financial situations becomes relatively stable, employing the expected value and conditional value at risk (conditional tail expectation) as the evaluation measures. We find the benefits and the contribution payments of the risk-sharing plan are not only at the level intermediate between the DB and DC plans because of the risk-sharing features, but also superior to them in some cases.

(2) Evaluation of design parameters of risk-sharing plan through sensitivity analysis
We conduct the sensitivity analysis of the five parameters to control the level of risk-sharing, and suggest how those parameters affect the plan design.

(3) Comparison of pension plans through the backtest
We implement the backtest using the historical data, and examine the actual effect on four pension plans for twenty years in Japan.

This paper is organized as follows. In Section 2, we sort out the concept of risk-sharing in order to design the plan. We show the calculation of four plans to evaluate them quantitatively; DB, DC, CB plan, and risk-sharing plan which is called 'RS plan', hereafter. In addition, we explain how to calculate the evaluation measures. In Section 3, we conduct the numerical analysis using the Monte Carlo simulation approach. We compare the outcomes of the RS plan with those of conventional pension plans to clarify the characteristics of risk-sharing. We also conduct the sensitivity analysis of the five parameters above. In Section 4, we implement the backtest in order to examine the actual effect on the pension plan. We compare the four plans using the historical data. Section 5 provides our concluding remarks.

## 2. Models concerning Corporate Pension Plan

We explain specific models of four kinds of corporate pension plans; DC, DB, CB, and RS plans.

### 2.1. Overview: Plan Design

#### 2.1.1. Basic Concept of Risk-sharing

We sort out the concept of risk-sharing to design the plan. We need to manage four kinds of risks shared by stakeholders; investment risk, longevity risk, interest rate risk, and inflation risk. Due to the historical background, many Japanese occupational pensions do not take longevity and inflation risks, and therefore we exclude them from shared risks. It is recognized that the interest rate risk is shared between a sponsoring company and participants/retirees through CB plan. We examine the effect of sharing interest rate risk by incorporating interest rate change in the plan design. Therefore, we design the structure where investment risk is only shared by each stakeholder based on CB plan, and examine the effect.

The stakeholders concerning risk-sharing are a sponsoring company, participants, and retirees. We assume we do not include nation (government), shareholders and so on.

#### 2.1.2. Assumption

The same amounts of salary are set in all plans. The amounts of salary of all participants are the same each other at the same point in time, and the real value is 1 for each participant. The salary increase rate over time is the same as the inflation rate. The benefit is received and the contribution is paid at the beginning of each year.

We need to match the size of all plans to compare them. At first we calculate the initial fund for paying benefit based on the DB plan so that each plan can pay a unit of defined benefit over payment periods from retirement age, and we set it as the unified target benefit level. However, the real value of assumed defined benefit at retirement age is one unit, and the nominal value of benefit paid after retirement is the same as the amount at retirement age. This reflects the standard design of occupational pensions in Japan.

The normal contributions need to be paid to accumulate funds for benefit at retirement age in all plans. The expected real yield of 10-year government bond is used as the discount rate for funding purpose and the contributions are constant on a real basis. We need to prepare the hypothetical account balances in CB and RS plans which consist of the pay credit and interest credit. The pay credit is the same as the normal contribution, and the interest credit is calculated using 10-year government bond yield. The fraction of deficiency/surplus is handled by adjusting the benefit, accrued liability and amortization. The plan asset is evaluated by the market value.

Next, we explain how to pay benefit. As mentioned above in the DB plan, the real benefit is 1 at retirement age, and the corresponding nominal defined benefit is fixed in the finite period. On the other hand, the fund at retirement is divided by beneficiary periods equally, and the adjusted amount of benefit by adding interest income is paid at each time. An interest rate used in the DC plan is an investment return after deduction of management fee. The interest rate used in the CB and RS plans is a government bond yield.[1]

We suppose the determined fraction (ex. 20%) of the deficiency to the funding standard, or the actuarial liability minus the plan asset in the DB and CB plans is amortized at the beginning of period. The amortization of funding deficiency is reset annually. Even when the deficiency is recognized next year again, the same fraction of deficiency needs to be amortized. When the surplus is recognized, the amortization becomes zero, but the normal contribution is paid. However, the normal contribution is not paid when the funding ratio is over 150%, which is called '150% rule' hereafter in this paper.

We evaluate the plan asset based on the market value. We adopt the constant rebalance strategies of asset allocation. We assume that the management fees deducted from the plan asset are 1.5% in the DC plan, and 0.5% in the DB, CB and RS plans in consideration of actual practice in Japan.

A pension benefit is paid annually, but not paid in the form of a lump-sum payment from a viewpoint of intergenerational risk adjustment. It should be noted that this assumption is not consistent with the actual practice of occupational pensions in Japan which is originated in the retirement allowance scheme, where a lump-sum payment is allowed instead of annual payment.

We summarize the design of DC, DB and CB plans in Table 1.[2]

Table 1: Design of DC, DB and CB plans

| | DC plan | DB plan | CB plan |
|---|---|---|---|
| Actuarial liability | Equal to plan asset | Calculate based on expected yield of 10-year government bond | Calculate based on real yield of 10-year government bond |
| Benefit | Calculate based on actuarial liability | Real benefit = 1 at retirement age, and nominal benefit is fixed in benefit period | Calculate based on actuarial liability |
| Contribution | Normal contribution | Normal contribution (with 150% rule) + Amortization | Normal contribution (with 150% rule) + Amortization |

[1] This situation of CB plan is different from the actual practice in Japan. However, this simplifies how to manage the hypothetical account balance using the benefit and interest credit at each time.

[2] The mathematical expressions are described in Appendix A.

#### 2.1.3. Method of risk-sharing

We build the structure of design and management of pension plan concerning risk-sharing based on the CB plan. The stakeholders concerning risk-sharing consist of a sponsoring company, active participants, and retirees. At first, we set two kinds of the funding ratios which trigger the risk-sharing; minimum funding ratio which triggers the deficiency-sharing, and maximum funding ratio which triggers the surplus-sharing. The example of the former is 105%, and that of the latter is 130%. The deficiency is shared by stakeholders in accordance with the sharing rule when the funding ratio is under the minimum ratio, and the surplus is shared when the funding ratio is over the maximum ratio. We need to determine the fraction when the deficiency or surplus is shared. For example, we suppose the sponsoring company shares the fraction of the deficiency/surplus with the active participants and retirees, and amortize its portion annually. The participants and retirees share the rest in proportion with the actuarial liability, and we also build the structure of risk-sharing between participants and retirees. The benefit decreases/increases by the actuarial liability multiplied by the fraction of the deficiency/surplus shared to the retirees. On the other hand, the actuarial liabilities of participants decrease/increase by the fraction of the deficiency/surplus shared to the participants.

In the RS plan, we do not set the 150% rule applied in the DB and CB plans. The minimum and maximum funding ratios concerning risk-sharing and the parameters with respect to the rule sharing deficiency/surplus the stakeholders bear/receive are fixed through the simulation period. The deficiency from the minimum ratio and the surplus from the maximum ratio are calculated based on the actuarial liability of the CB plan. The benefit adjusted in the former year does not carry forward to the following year.

#### 2.1.4. Setting

An age composition of the insured persons is time-homogeneous in the simulation period. For simplicity, the decrement rate and mortality rate are not considered. The number of persons of each age is assumed to be 1 from 20 to 79 years old. The working period is under 65 years old, and the payment period for pension benefit is over 65 years old. For convenience, we shift 20 years backward in the after-mentioned calculation, and the total period is from 0 to 59 years old.

It is supposed that the initial actuarial liability is set in the simulation, using the guaranteed rate under the static population. The initial actuarial liability of active participants in the DB plan is the same as that in the CB plan due to no consideration of early retirement, whereas that of retirees in the DB plan is slightly smaller than that in the CB plan due to the different pension payments. We assume the initial plan assets in the DB, CB and RS plans are the same as the actuarial liabilities, respectively. Therefore, the statuses of the DB and CB plans are fully funding, and the deficit/surplus of each plan is zero. On the other hand, the risk-sharing mechanism is immediately triggered in the RS plan if the minimum funding ratio is over 1. The initial actuarial liability in the DC plan is the same as that in the CB plan. The plan asset and its allocation to each individual account are the same as the actuarial liability in the DC plan.

#### 2.1.5. Notations

A working period (funding period) is from 0 to $T_L - 1$ years old, and a benefit period is from $T_L$ to $T_L + T_R - 1$ years old, where $T_L$ is the number of years of working period, and $T_R$ is the number of years of benefit period. We suppose the number of participants/retirees of $x$ years old at time $n$ is $l_{x,n} = 1$. The nominal salary of worker is 1 at time 1, and it is calculated as $\exp\left(\sum_{k=1}^{n-1} i_k^{(s)}\right)$ at time $n$, where $i_k^{(s)}$ is the inflation rate at time $k$. This means that the real salary of worker is 1 at any time. Actuarial liability at retirement divided by $T_R$ periods equally is assigned to the original amount of benefit, and the amount adjusted by interest income is paid at each time. The fund is invested with constant rebalance strategy where the portfolio weight is fixed.

We evaluate the benefits and contributions using Monte Carlo simulation approach. The notations are as follows. We put '$(s)$' to the notations to express the discretized distribution using multiple random samples. The bar denotes the expected value. The dash denotes the nominal value, while the notations without the dash denote the real values which are adjusted by the inflation rate.

$Q$ : Number of samples

$T_L$ : Number of years of working period

$T_R$ : Number of years of payment period for pension benefit

$N$ : Number of years of simulation period

$m$ : Management fee

$i_n^{(s)}$ : Inflation rate of sample $s$ in period $n$

$\overline{I}$ : Expected inflation rate

$j_n^{(s)}$, $j_n^{(s)\prime}$ : Real/Nominal 10-year government bond yield of sample $s$ in period $n$ $\quad j_n^{(s)} = j_n^{(s)\prime} - i_n^{(s)}$

$\overline{J}$, $\overline{J}'$ : Expected real/nominal 10-year government bond yield $\quad \overline{J} = \overline{J}' - \overline{I}$

$U$ : Number of assets

$r_{u,n}^{(s)}$, $r_{u,n}^{(s)\prime}$ : Real/Nominal rate of return of asset $u$ in period $n$ $(u = 1, \ldots, U)$ $\quad r_{u,n}^{(s)} = r_{u,n}^{(s)\prime} - i_n^{(s)}$

$\overline{R}_u$, $\overline{R}_u'$ : Expected real/nominal rate of return of asset $u$ $(u = 1, \ldots, U)$ $\quad \overline{R}_u = \overline{R}_u' - \overline{I}$

$w_u$ : investment weight of asset $u$ $(u = 1, \ldots, U)$

$r_{M,n}^{(s)}$, $r_{M,n}^{(s)\prime}$ : Real/Nominal rate of return of portfolio $M$ in period $n$

$\quad r_{M,n}^{(s)\prime} = \sum_{u=1}^{U} w_u r_{u,n}^{(s)\prime}$, $r_{M,n}^{(s)} = r_{M,n}^{(s)\prime} - i_n^{(s)} = \sum_{u=1}^{U} w_u r_{u,n}^{(s)}$

$\overline{R}_M$, $\overline{R}_M'$ : Expected real/nominal rate of return of portfolio $M$ $\quad \overline{R}_M = \overline{R}_M' - i_n^{(s)} = \sum_{u=1}^{U} w_u \overline{R}_u$

$p^1$ : Normal contribution rate (commonly utilized to all pension plans)

The mathematical expressions of the existing plans (DC, DB and CB plans) are omitted due to space limitation.

## 2.2. Risk-sharing plan

We share investment risk based on the actuarial liability $L_n^{(s)}$ and benefit $B_n^{(s)}$ calculated for the CB plan. The normal contribution of the CB plan can be zero due to the 150% rule, but we assume the normal contribution of RS plan is constant because we adjust the contribution by the amortization.

We incorporate the following five parameters to control the level of risk-sharing, where $F_n^{(s)}$ is a plan asset. The parameters consist of two kinds of trigger parameters and three kinds of sharing parameters.

Trigger parameters

$T^{(1)}$ : minimum funding ratio which triggers the deficiency-sharing ( $T^{(1)} \geq 1$)
— deficiency of plan asset to funding target $T^{(1)} L_n^{(s)}$: $U_n^{(s)} = \max(T^{(1)} L_n^{(s)} - F_n^{(s)}, 0)$

$T^{(2)}$ : maximum funding ratio which triggers the surplus-sharing ($T^{(2)} \geq T^{(1)}$)
— surplus of plan asset to funding target $T^{(2)} L_n^{(s)}$: $S_n^{(s)} = \max(F_n^{(s)} - T^{(2)} L_n^{(s)}, 0)$

Sharing parameters

$K^{(0)}$ : fraction of the deficiency/surplus the sponsoring company will bear/receive ($0 \leq K^{(0)} \leq 1$)

$K^{(1)}$ : annual amortization/decumulation ratio of the deficit/surplus allocated to the sponsoring company ($0 \leq K^{(1)} \leq 1$) [3]

$K^{(2)}$ : fraction of deficiency/surplus the retirees will share with active participants ($0 \leq K^{(2)} \leq 1$)

The negative amortization is paid to a sponsoring company who receives the surplus or the upside

[3]The definition is the same as the parameter used in DB and CB plans.

deviation from $T^{(2)}L_n^{(s)}$. This means that it is paid back to a sponsoring company, participants and retirees in accordance with the risk-sharing rule [4]. We suppose the risk is shared with participants fairly by employing the fraction $K^{(2)}$ which is the same as the fraction in bearing deficiency.

Pension financing of the RS plan is evaluated based on normal contributions, actuarial liabilities, benefits of the CB plan. The deficiency/surplus is adjusted annually, and therefore the fraction of funding deficiency/surplus which can be amortized is constant, denoted by $K^{(1)}$.

We decide the sharing method depending on $U_n^{(s)}$ and $S_n^{(s)}$ at time $n$. The hat ($\hat{}$) is put to show the actual amount adjusted in sharing risk. We calculate the normal contribution in period $n$ as $C_n^{1(s)} = T_L \cdot p^1$. We introduce a new notation $Z_n^{(s)}$ in order to express $S_n^{(s)}$ and $U_n^{(s)}$ simultaneously as follows.

$$Z_n^{(s)} = \begin{cases} T^{(1)}L_n^{(s)} - F_n^{(s)} & (\frac{F_n^{(s)}}{L_n^{(s)}} < T^{(1)}) \\ 0 & (T^{(1)} \le \frac{F_n^{(s)}}{L_n^{(s)}} \le T^{(2)}) \\ -(F_n^{(s)} - T^{(2)}L_n^{(s)}) & (\frac{F_n^{(s)}}{L_n^{(s)}} > T^{(2)}) \end{cases}$$

$Z_n^{(s)}$ is the deficiency of the plan asset from the funding target $T^{(1)}L_n^{(s)}$ for $Z_n^{(s)} > 0$, while $Z_n^{(s)}$ is the surplus from the funding target $T^{(2)}L_n^{(s)}$ for $Z_n^{(s)} < 0$. The amount of deficiency is born, or that of surplus is received by stakeholders. The amount is paid/born when $Z_n^{(s)} > 0$, and received when $Z_n^{(s)} < 0$. The amount paid/received by a sponsoring company is $K^{(0)}Z_n^{(s)}$. The amount born/received by participants and retirees is $(1 - K^{(0)})Z_n^{(s)}$, and it is separated as follows:

$$Z_n^{a(s)} = (1 - K^{(0)}) \left(1 - \frac{K^{(2)}L_n^{p(s)}}{L_n^{(s)}}\right) Z_n^{(s)} \text{ for participants,} \tag{2.1}$$

$$Z_n^{p(s)} = (1 - K^{(0)}) \left(\frac{K^{(2)}L_n^{p(s)}}{L_n^{(s)}}\right) Z_n^{(s)} \text{ for retirees.} \tag{2.2}$$

where $L_n^{p(s)}$ is the sum of the actuarial liabilities of retirees at period $n$. The sponsoring company pays/receives the amortization as,

$$C_n^{2(s)} = K^{(0)}K^{(1)}Z_n^{(s)}. \tag{2.3}$$

The contributions of the RS plan are dependent on the funding ratio, and calculated as,

$$C_n^{(s)} = \begin{cases} T_L \cdot p^1 + \left(T^{(1)}L_n^{(s)} - F_n^{(s)}\right) K^{(0)}K^{(1)} & (F_n^{(s)} < T^{(1)}L_n^{(s)}) \\ T_L \cdot p^1 & (T^{(1)}L_n^{(s)} \le F_n^{(s)} < T^{(2)}L_n^{(s)}) \\ T_L \cdot p^1 - \left(F_n^{(s)} - T^{(2)}L_n^{(s)}\right) K^{(0)}K^{(1)} & (F_n^{(s)} \ge T^{(2)}L_n^{(s)}) \end{cases} \tag{2.4}$$

The relationship is shown in Figure 1.

The actual actuarial liability and benefit are calculated for each year as,

$$\hat{L}_n^{(s)} = L_n^{(s)} - (1 - K^{(0)})Z_n^{(s)} \tag{2.5}$$

$$\begin{aligned} \hat{B}_n^{(s)} &= \left(1 - \frac{Z_n^{p(s)}}{L_n^{p(s)}}\right) B_n^{(s)} \\ &= \begin{cases} B_n^{(s)} \left\{1 - (1 - K^{(0)})K^{(2)} \cdot \left(T^{(1)} - \frac{F_n^{(s)}}{L_n^{(s)}}\right)\right\} & (\frac{F_n^{(s)}}{L_n^{(s)}} < T^{(1)}) \\ B_n^{(s)} & (T^{(1)} \le \frac{F_n^{(s)}}{L_n^{(s)}} \le T^{(2)}) \\ B_n^{(s)} \left\{1 + (1 - K^{(0)})K^{(2)} \cdot \left(\frac{F_n^{(s)}}{L_n^{(s)}} - T^{(2)}\right)\right\} & (\frac{F_n^{(s)}}{L_n^{(s)}} > T^{(2)}) \end{cases} \end{aligned} \tag{2.6}$$

[4] We do not employ the rule that the surplus is given back to the sponsoring company, because we would like to employ the rule that the deficiency can be amortized, and participants and retirees cannot receive it at a time.

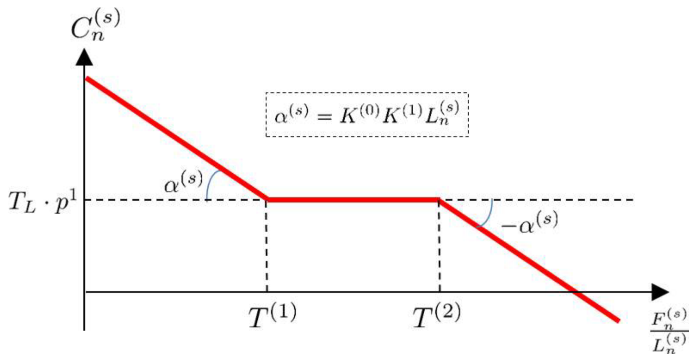


Figure 1: The relationship between contributions and funding ratio(RS plan)

The actual actuarial liability is calculated by adjusting the amounts to the actuarial liability of CB plan which are paid/received by a sponsoring company, participants, and retirees. The actual benefits are adjusted as well. Equation (2.6) can be depicted as in Figure 2.

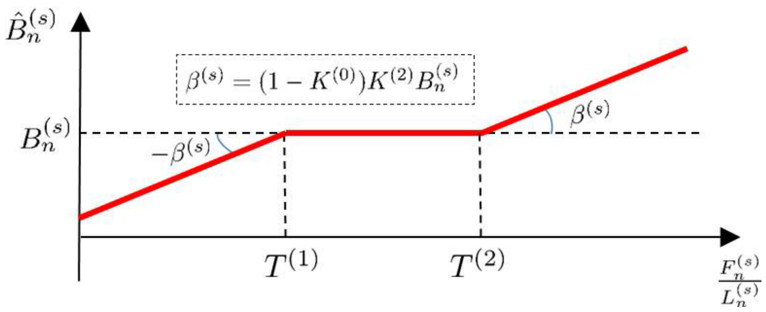


Figure 2: The relationship between benefits and funding ratio(RS plan)

The plan asset is calculated as follows.

$$F_1^{(s)} = L_1^{(s)} \tag{2.7}$$

$$F_n^{(s)} = \left(F_{n-1}^{(s)} + C_{n-1}^{(s)} - \hat{B}_{n-1}^{(s)}\right) \exp\left(r_{M,n-1}^{(s)} - m\right), \ (n = 2, \ldots, N) \tag{2.8}$$

We can express the adjusted fraction of amortization for a sponsoring company and benefit to funding deficiency/surplus for retirees by separating the paid/received fraction for a sponsoring company, participants and retirees as in Figure 3.

### 2.3. Evaluation of utility

#### 2.3.1. Methodology

We evaluate the utilities based on the following condition.

- We use $N - N'$ years in the latter period to evaluate the utility under the steady-state condition, where $N$-year period is the simulation period and the former $N'$-year period is exempted. This is because the distributions in the former period are dependent on the initial deterministic value. We set $N = 100$ and $N' = 40$ in the numerical analysis of Section 3.
- A sponsoring company, participants, and retirees share investment risk. Participants and retirees evaluate pension plans using the mean and CVaR of benefit. A sponsoring company evaluates them using those of contribution which consists of the normal contribution and amortization.
- We consider that it is more important to evaluate time-series state variability on each path than variability of states at specific time. It is better for the benefits not to fluctuate on the downside, and the larger expected benefit is better. On the other hand, it is better for the contributions not to fluctuate on the upside, and the smaller expected contribution is better.

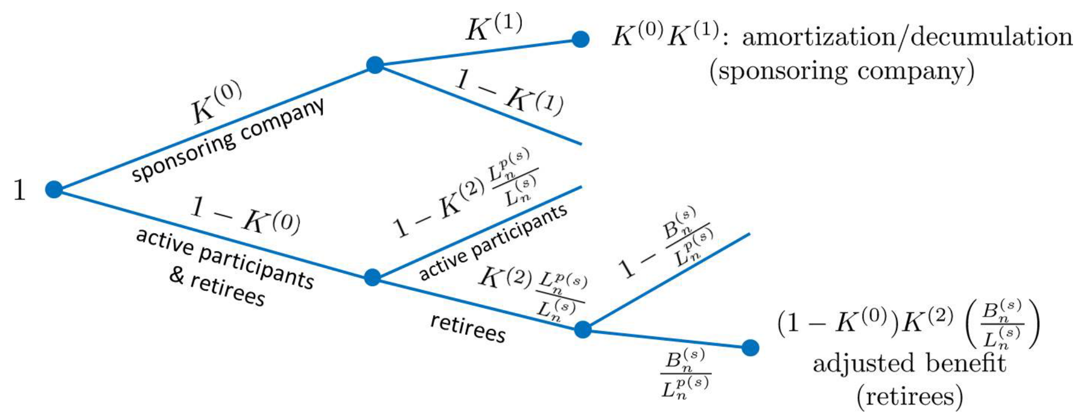


Figure 3: Adjusted fraction of amortization and benefit to the funding deficiency/surplus

### 2.3.2. Evaluation measures

At first, we define a random variable at period $n$ as $\tilde{x}$ which shows the amount of benefit or contribution. We define a random variable $\tilde{u}_1$ which shows the time-series mean and $\tilde{u}_2$ which shows the time-series CVaR using the above-mentioned random variable $\tilde{x}$ as follows.

$$\text{time-series mean} \;:\; \tilde{u}_1 = \frac{\sum_{n=N'+1}^{N} \tilde{x}_n}{N-N'},$$

$$\text{time-series CVaR} \;:\; \tilde{u}_2 = \text{ts-CVaR}(\beta) = \frac{1}{(1-\beta)(N-N')} \sum_{n=N'+1}^{N} \mathbf{1}_{\{\tilde{x}_n < \text{ts-VaR}_x(\beta)\}} \tilde{x}_n,$$

where $\tilde{x}_n$ is a random variable which is better for a larger value, and we define ts-VaR$(\beta) = \max\{X \,|\, \Pr(X \leq \tilde{x}_n) \leq 1-\beta\}$. We adopt the mean of $\tilde{u}_1$ and CVaR of $\tilde{u}_2$ as evaluation measures. We can show the mean and CVaR as in Figure 4. The larger mean and CVaR of the benefit are better for retirees, whereas the smaller mean and CVaR of the contribution are better for a sponsoring company.

The graphs on the top of Figure 4 are time-series variation, and we have $Q$ graphs in the Monte Carlo simulation. The middle graphs show the mean and the bottom graphs show the CVaR. The CVaR of benefit is located in the downside of the distribution, and the CVaR of contribution is located in the upside.

## 3. Numerical Analysis

We examine the risk-sharing plan by the numerical analysis using the Monte Carlo simulation approach. We compare the outcomes of the risk-sharing plan with those of three pension plans; DC, DB and CB plans. We also conduct the sensitivity analysis of the five parameters of risk-sharing plan, and highlight the characteristics of the risk-sharing functions through various simulation analyses.

### 3.1. Setting

The parameters are as follows.

- Number of years of working period is $T_L = 45$.
- Number of years of payment period for pension benefit is $T_R = 15$.
- Number of years of simulation period is $N = 100$.
- Number of years of exemption period is $N' = 40$.
- Management fee of DC plan is $m$ =150bp (1.5%), and that of other plans is $m$ =50bp (0.5%).
- Confidence level of CVaR: $\beta = 0.95$

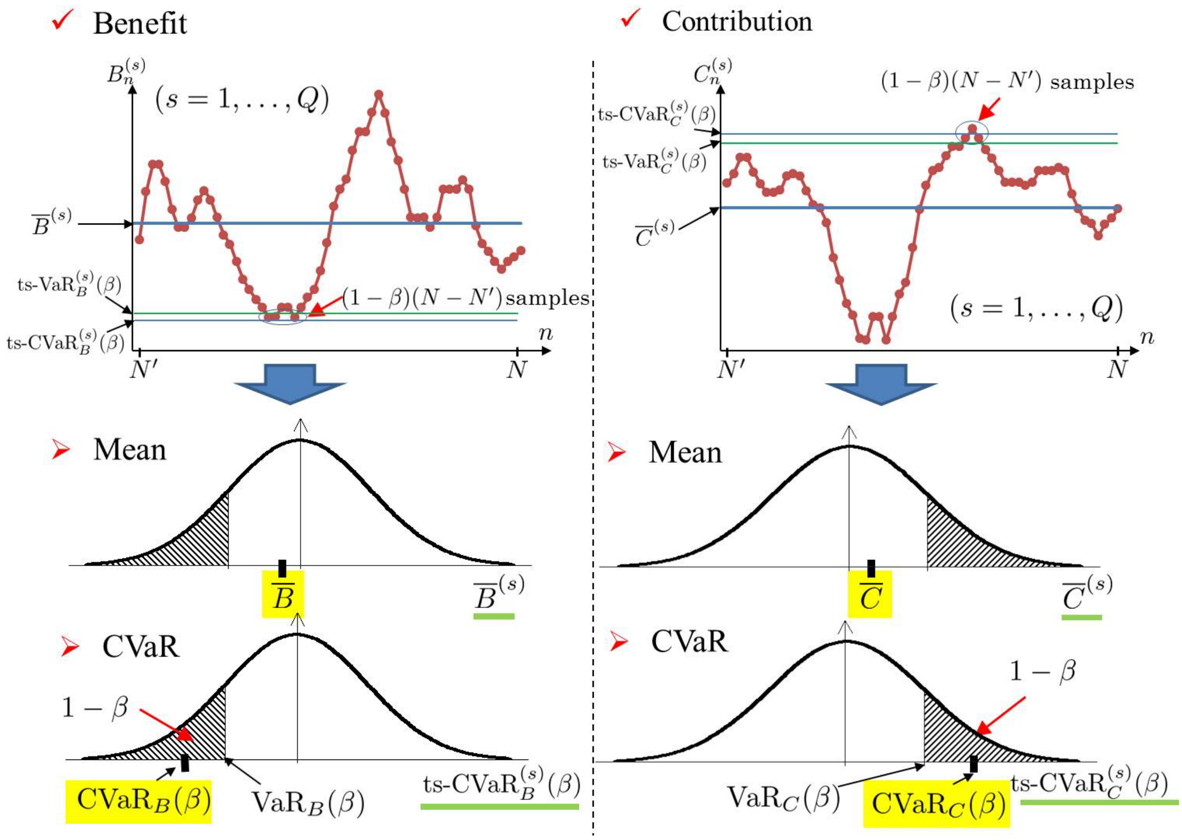


Figure 4: Evaluation measures of benefit and contribution

- We generate random samples of the nominal rate of return of five assets (domestic stock and bond, foreign stock and bond, and cash), long-term bond yield, and inflation rate.
- We show the expected rate of return, standard deviation, and correlation in Table 2. Except the expected rate of return of domestic bond and 10-year government bond yield, those are parameters used in order to derive the new policy asset mix for the third medium-term plan published by Government Pension Investment Fund (GPIF)[6]. We use the wage growth rate as the inflation rate. We assume the expected rate of return of 10-year government bond yield is the same as that of domestic bond, and the correlations between 10-year government bond yield and other assets are the same as those between short-term asset and other assets. We assume that the coefficient of variation is the same as that of short-term asset, and therefore the standard deviation of 10-year government bond yield is set to 1.5%($\approx 3.4 \times 0.5/1.1$).
- The time-series correlations of assets are zero. The random samples are generated based on the parameters of normal distribution in Table 2. The number of scenarios is 10,000 over a hundred years. ($Q = 10,000$)
- Additional base parameters of risk-sharing plan are $T^{(1)} = 1.05$, $T^{(2)} = 1.3$, $K^{(0)} = 0.5$, $K^{(1)} = 0.2$, and $K^{(2)} = 0.5$.
- Additional base parameter of DB and CB plans is $K^{(1)} = 0.2$.
- We examine the combination of three kinds of expected rates of return (Return (A), (B), and (C)) and two kinds of portfolios (Portfolio [a] and [b]). For example, we call the combination of Return (A) and Portfolio [a] 'Case Aa'.

Table 2: Expected return, standard deviation and correlation

| | DS | DB | FS | FB | SA | IR | 10Y-GB |
|---|---|---|---|---|---|---|---|
| Expected return | 6.0% | 3.4% | 6.4% | 3.7% | 1.1% | 2.8% | 3.4% |
| Standard deviation | 25.1% | 4.7% | 27.3% | 12.6% | 0.5% | 1.9% | 1.5% |
| Correlation | DS | DB | FS | FB | SA | IR | 10Y-GB |
| Domestic stock (DS) | 1.00 | −0.16 | 0.64 | 0.04 | −0.10 | 0.12 | −0.10 |
| Domestic bond (DB) | −0.16 | 1.00 | 0.09 | 0.25 | 0.12 | 0.18 | 0.12 |
| Foreign stock (FS) | 0.64 | 0.09 | 1.00 | 0.57 | −0.14 | 0.10 | −0.14 |
| Foreign bond (FB) | 0.04 | 0.25 | 0.57 | 1.00 | −0.15 | 0.07 | −0.15 |
| Short-term asset (SA) | −0.10 | 0.12 | −0.14 | −0.15 | 1.00 | 0.35 | 1.00 |
| Inflation rate (IR) | 0.12 | 0.18 | 0.10 | 0.07 | 0.35 | 1.00 | 0.35 |
| Government bond (10Y-GB) | −0.10 | 0.12 | −0.14 | −0.15 | 1.00 | 0.35 | 1.00 |

Three kinds of expected rates of return for sensitivity analysis

| | DS | DB | FS | FB | SA | IR | 10Y-GB |
|---|---|---|---|---|---|---|---|
| Return (A) | 6.0% | 3.4% | 6.4% | 3.7% | 1.1% | 2.8% | 3.4% |
| Return (B) | 3.0% | 1.7% | 3.2% | 1.85% | 0.55% | 1.4% | 1.7% |
| Return (C) | 0.0% | 3.4% | 0.0% | 3.7% | 1.1% | 2.8% | 3.4% |

※ Return (A): Table 2, Return (B): 0.5 × Return (A),
Return (C): 0% expected rate of return of stocks in Return (A)

Two kinds of portfolios for sensitivity analysis

| | DS | DB | FS | FB | Stock:Bond |
|---|---|---|---|---|---|
| Portfolio [a] | 25% | 35% | 25% | 15% | 5:5 |
| Portfolio [b] | 5% | 55% | 5% | 35% | 1:9 |

Expected real rate of return and standard deviation

| | Expected real rate of return | | | Standard deviation |
|---|---|---|---|---|
| | Return (A) | Return (B) | Return (C) | |
| Portfolio [a] | 2.045% | 1.0225% | −1.055% | 12.766% |
| Portfolio [b] | 0.985% | 0.4925% | 0.365% | 6.661% |

### 3.2. Base analysis

It is assumed that four plans have almost the same initial actuarial liabilities for comparison. We observe the distributions of actuarial liabilities for a hundred years. Though we omit the graphs due to space limitation, we find they are dependent on the initial value in the early periods, but they are gradually close to being time-homogeneous as time passes.

We examine the results of Case Aa as a base case. We show the seven kinds of percentiles (1%, 5%, 25%, 50%, 75%, 95%, 99%) of the distributions of benefit and contribution in Figures 5 and 6.

At first, we examine the characteristic of the distribution of benefit in Figure 5. We find the variability of the DB plan is smaller than those of other plans. The reason is that the benefit of DB plan is affected by an inflation rate, but the amount of benefit is fixed to 1 at retirement, and the variability of the sum of benefits becomes small. The ascending order of the variabilities after a hundred years is as follows: DB, CB, RS, and DC plans. The variability of the benefit of the DC plan is larger than that of the CB plan because the standard deviation of the rate of return of the portfolio used calculating the benefit of the DC plan is larger than that of the government bond yield used calculating the benefit of the CB plan. The variability of benefit of DC plan is larger than that of the RS plan because the volatility of the rate of return of the portfolio is larger than the change of benefit based on the funding ratio. The variability of the benefit of the RS plan is larger than that of the CB plan because the benefit of the RS plan is basically the same as that of the CB plan, but is dependent on the adjustment determined by the funding ratio[5].

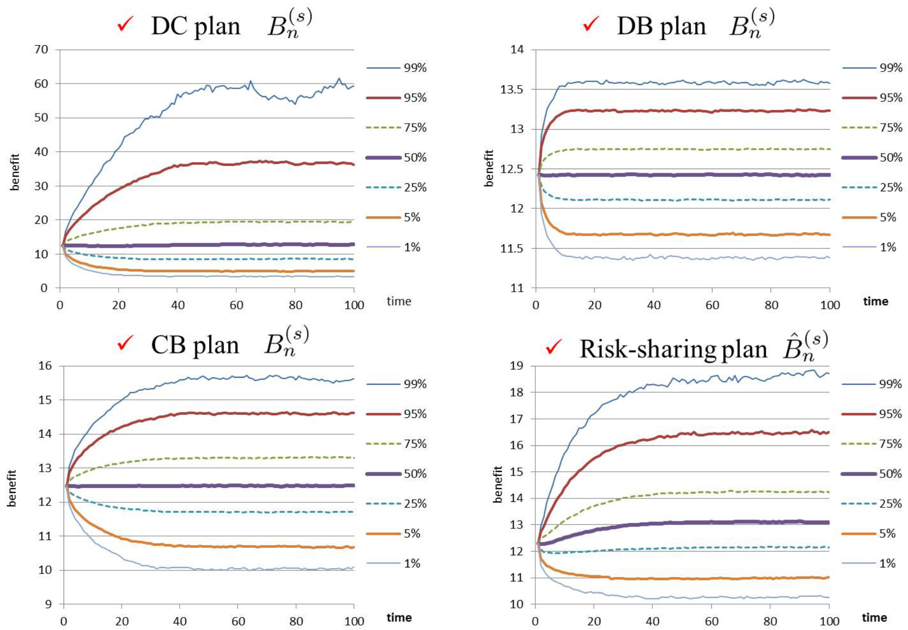


Figure 5: Benefit (Case Aa)

Next, we examine the characteristics of the distribution of contribution in Figure 6. The contribution of the DC plan is constant, and therefore we discuss other three plans. The shapes of distributions of amortizations of DB and CB plans are similar because they are dependent on the funding ratio. The contributions of the DB and CB plans do not become negative, but those of the RS plan become negative when the funding ratio is larger than the threshold, and it is possible to decrease cost of a sponsoring company. The ascending order of the variabilities of distributions except the DC plan is DB, CB, and RS plans, while that of the 99th percentile is RS, DB, and CB plans. The sensitivity of the contribution to the funding ratio in the DB and CB plans is 0.2 times actuarial liability when the funding ratio is less than 1. When the funding ratio is larger than 1.5, the contribution is equal to zero. Meanwhile, as shown in Figure 1, the sensitivities of the contribution to the funding ratio in the RS plan is 0.1 times actuarial liability when the funding ratio is less than 1.05, and $-0.1$ times actuarial liability when the funding ratio is larger than 1.3, respectively. The contributions of the RS plan are not likely to become larger than those of the DB and CB plans because the sensitivity of the RS plan is smaller when the funding ratio is less than the lower threshold. Meanwhile, they are likely to become smaller than those of the DB and CB plans when the funding ratio is more than the upper threshold. However, we note that we cannot compare them exactly because of different thresholds.

Based on the results shown in Figures 5 and 6, the distributions of benefits and contributions become stable when about forty years pass, and we determine that the exemption period is forty years ($N' = 40$), and evaluate the means and CVaRs. We show the means and CVaRs of benefits and contributions of four plans for the six cases in Figure 7. The six cases are the combination of three kinds of expected return and two kinds of portfolios.

The larger mean and CVaR of benefit are better, and the smaller mean and CVaR of contribution are better. Therefore, the benefit becomes better toward upper right side in the diagram, and the contribution becomes better toward lower left side. The benefits of the DB and CB plans

[5]The percentiles of benefit of the RS plan are larger than those of the CB plan after a hundred years. This means that the benefit of the RS plan stochastically increases more than those of the CB plan.

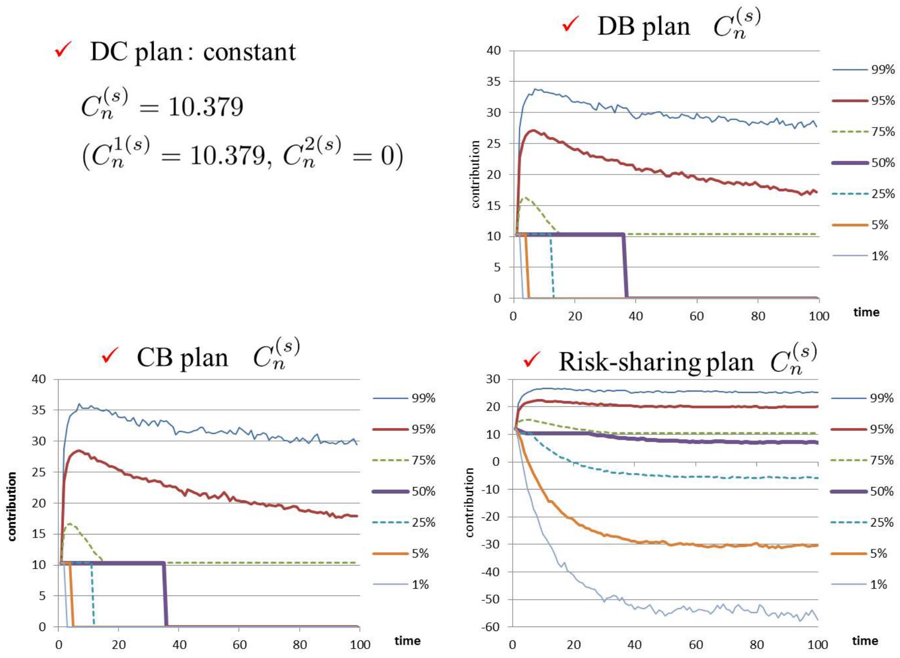


Figure 6: Contribution (Case Aa)

on the upper graphs are the same as those on the lower graphs because they are independent of portfolios[6]. The contribution of the DC plan is deterministic, and therefore we apply the value to the mean and CVaR.

The benefit of the RS plan and the contributions of the DB, CB and RS plans are dependent on the funding ratio. When the portfolio return becomes large, the plan asset becomes large. Therefore the benefit becomes large and the contribution becomes small. We sort six cases in descending order of the expected rate of portfolio return, and we sort four plans in ascending order of the mean benefit and contribution in the respective case, and we show the relationship in Table 3.

As the expected rate of return becomes small, the mean benefit of the DC plan becomes smaller, compared with other plans. This is because the DC plan is easily affected by the portfolio return. In addition, the management fee of DC plan is 150 bp, but those of other plans are 50bp, and therefore the return after deduction of management fee of DC plan is likely to become smaller than those of other plans. The relationship of DB $<$ CB holds for all cases. This results from the difference between the benefit designs in the payment period. If both pension plans have the same benefit resourses, the benefits of DB plan which nominal values are fixed are larger than those of CB plan in the former payment period. Therefore the amounts of benefits in the CB plan are larger than those in the DB plan in the latter payment period, and the total amounts in the CB plan are larger than those in the DB plan. The benefit of the RS plan is easily affected by the funding ratio, compared with the DB and CB plans, and it becomes small as the expected rate of return becomes small.

The relationship of RS $<$ DB $<$ CB holds for the mean contributions. The mean contributions of the DC plan are the largest in the cases of Aa, Ba and Ab where the expected rates of return are relatively large, whereas those are the smallest in the cases of Bb, Cb and Ca where the expected rates of return are relatively small. The DC plan is relatively less advantageous under the better investment opportunity because the contribution of DC plan is deterministic, and those of other

[6]The benefit of the DB plan is dependent on the inflation rate and government bond yield, and the benefit of the CB plan is dependent on the government bond yield.

plans are dependent on the funding ratio. As above mentioned, the contribution of the RS plan becomes smaller than those of DB and CB plans because the amortization of RS plan is likely to become negative. The relationship of DB $<$ CB holds because the actuarial liabilities of retirees of DB plan is smaller.

We examine the relationship of CVaRs among the four plans for the six cases. The CVaR of benefit of the RS plan is almost the same as that of the CB plan regardless of the expected rate of return, however the relationship of DC $<$ (RS $\approx$ CB) $<$ DB holds. The relationship of DC $<$ RS $<$ DB $<$ CB holds for the contributions. The reason is as follows. The CVaR of benefit is the smallest because the downside risk of the DC plan asset of is the largest. The benefit of the DB plan is the most stable, and therefore the downside risk is the smallest or the CVaR is the largest. Those of the RS and CB plans are in between. On the other hand, the CVaR of contribution of the DC plan is the smallest because of the deterministic contribution. The reason that the relationship of RS $<$ DB $<$ CB for the CVaRs of contributions holds is the same as the reason for the mean contribution.

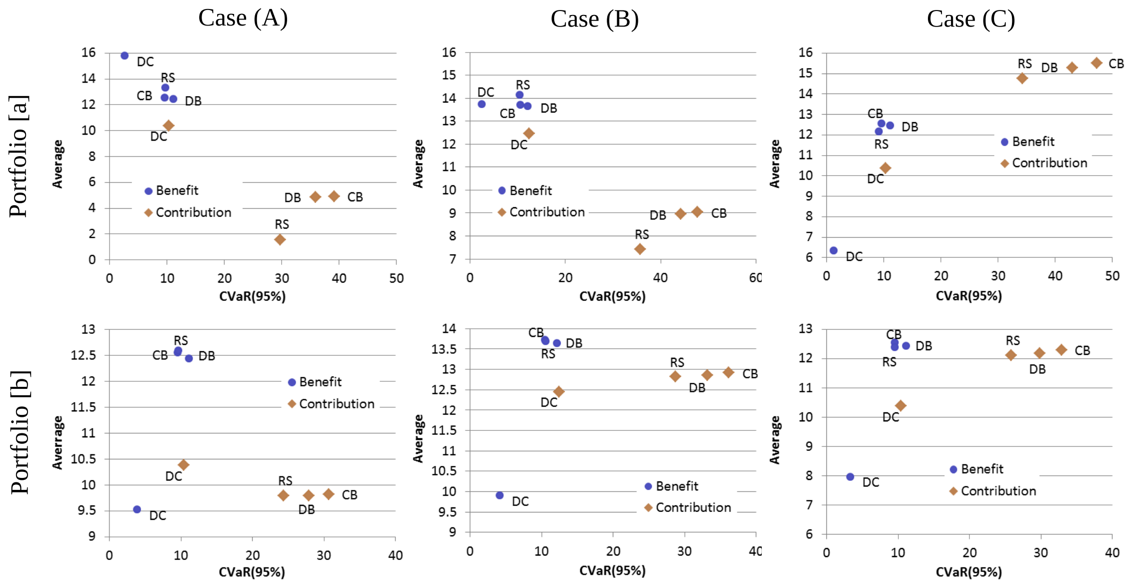


Figure 7: Comparisons of benefits and contributions of four plans for six cases

Table 3: Relationship of mean benefits and mean contributions

| case | expected return | mean benefit | mean contribution |
|---|---:|---|---|
| Aa | 2.0450% | DB $<$ CB $<$ RS $<$ DC | |
| Ba | 1.0225% | DB $<$ CB $<$ DC $<$ RS | RS $<$ DB $<$ CB $<$ DC |
| Ab | 0.9850% | DC $<$ DB $<$ CB $<$ RS | |
| Bb | 0.4925% | DC $<$ DB $<$ RS $<$ CB | |
| Cb | 0.3650% | DC $<$ RS $<$ DB $<$ CB | DC $<$ RS $<$ DB $<$ CB |
| Ca | $-1.0550$% | | |

### 3.3. Sensitivity analysis for the parameters of RS plan

We conduct the sensitivity analysis for the parameters of the RS plan. The values of parameters are as follows. The base parameters in Section 3.2 are underlined.

- Minimum funding ratio which triggers the deficiency-sharing: $T^{(1)} = 1$, $\underline{1.05}$, 1.1, 1.15, 1.2
- Maximum funding ratio which triggers the surplus-sharing: $T^{(2)} = 1.2$, $\underline{1.3}$, 1.4, 1.5, 1.6

- Fraction of the deficiency/surplus a sponsoring company will bear/receive: $K^{(0)} = 0,\ 0.05,\ 0.1,\ 0.15,\ 0.2,\ 0.25,\ \underline{0.5},\ 0.75,\ 1$
- Annual amortization/decumulation ratio of the deficiency/surplus allocated to a sponsoring company: $K^{(1)} = 0,\ 0.02,\ 0.04,\ 0.06,\ 0.08,\ 0.1,\ \underline{0.2},\ 0.4,\ 0.6$
- Fraction of deficiency/surplus the retirees will transfer to active participants: $K^{(2)} = 0, 0.25,\ \underline{0.5},\ 0.75,\ 1$

We show the means and CVaRs for the five parameters in Case Aa in Figure 8.

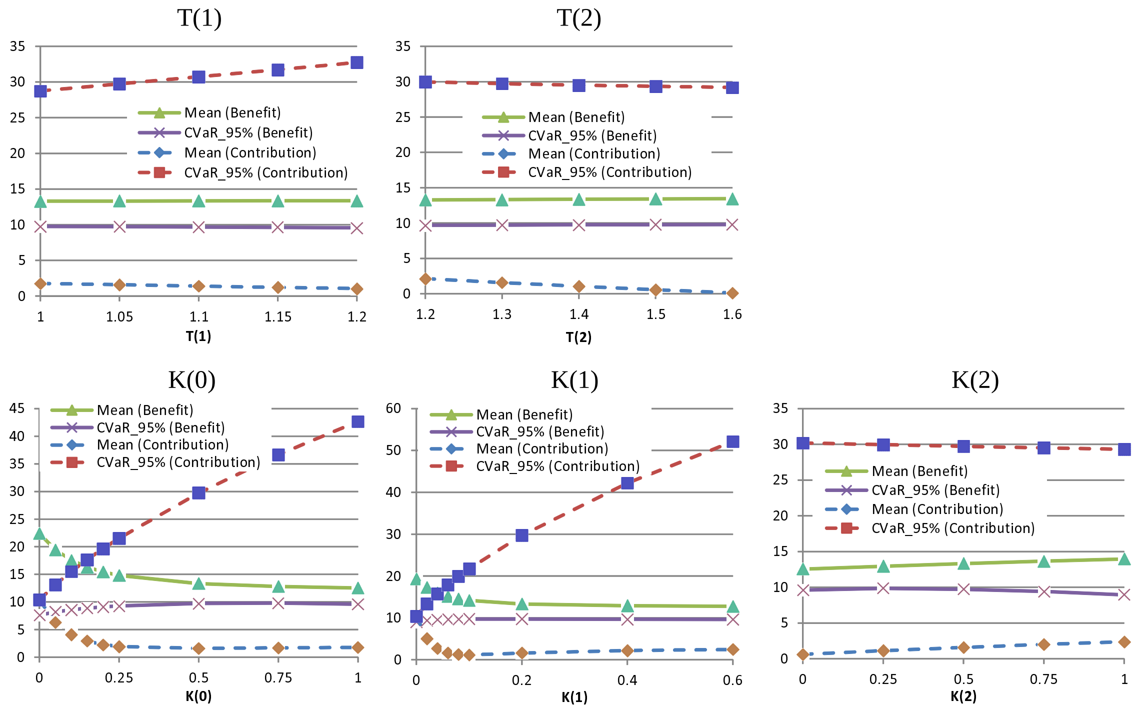


Figure 8: Sensitivity analysis of five parameters in Case Aa

The parameters $T^{(1)}$ and $T^{(2)}$ are related with the funding ratio which triggers the deficiency/surplus-sharing. The deficiency is easy to be recognized for a large $T^{(1)}$, and the surplus is not easy to be recognized for a large $T^{(2)}$. Therefore, the amortization is large and the benefit is small for a large $T^{(1)}$ or $T^{(2)}$. As the result, the plan asset is large, but it leads to the small amortization and large benefit. This means that the direction of the effect on the amortization and benefit are dependent on the parameter values. We may require the additional funding to the actuarial liability for large $T^{(1)}$. This affects the reduction of benefit due to funding deficiency. However it does not affect the results, according to the result of Figure 8. On the other hand, it also affects the increase in the amortization. We check the difference between the mean and CVaR of the contribution becomes large as $T^{(1)}$ becomes large as in Figure 8. The 20% of the deficiency/surplus are allocated to a sponsoring company, and it affects more than the benefit in the base analysis. The parameter $T^{(1)}$ affects the sponsoring company more than participants and retirees.

As $T^{(2)}$ is large, the level of funding becomes large, and therefore it is expected that the possibilities of the redution of the benefit and the increase in the amortization become small. In addition, it is also expected that the benefit and the decumulation from the surplus increase when the investment return is beyond the guaranteed rate of return. The large $T^{(2)}$ leads to a good outcome for both a sponsoring company and participants/retirees in the long term in the case where the interest profit is obtained as in Case Aa. However, these effects cannot be always expected in the case where the interest profit is obtained.

The parameters $K^{(0)}$, $K^{(1)}$, and $K^{(2)}$ are related with the deficiency/surplus a sponsoring company, active participants, and retirees bear/receive.

The parameter $K^{(0)}$ is associated with the risk-sharing of deficiency/surplus between a sponsoring company and participants/retirees, and there is mutually a trade-off and zero-sum relationship between them. When $K^{(0)} = 0$, a sponsoring company does not take on investment risk, but participants/retirees take on risk. As a result, the contribution is fixed because it consists of the normal contribution, but the benefit is volatile. When $K^{(0)} = 1$, the contribution is volatile, but the benefit is fixed. Therefore, as $K^{(0)}$ is large, the amortization is likely to change to the funding ratio, but the actual benefit is not likely to change. As shown in Figures 1 and 2, the sensitivity of the amortization to the funding ratio is proportion to the actuarial liability $L_n^{(s)}$, and the sensitivity of the benefit to the funding ratio is proportion to the benefit of CB plan $B_n^{(s)}$. The sensitivity of the contribution is larger than that of the benefit because the amount of actuarial liability is larger than the amount of benefit. The lower left graph of Figure 8 shows that the means of both benefit and contribution become small when $K^{(0)}$ is large. The reason is that these situations are likely to occur in Case Aa where the expected rate of return of the portfolio is large. Due to space limitation, we omit a result, but when $K^{(0)}$ is large, the means of both benefit and contribution become large in Case Ca where the expected return is small. This shows that the effects on the means are dependent on the expected return of the portfolio.

The graph also shows that the CVaR of the benefit becomes slightly large, and the CVaR of contribution becomes large when $K^{(0)}$ is large. The reason is that the sensitivities of amortization and benefit to the funding ratio are dependent on $K^{(0)}$, and the CVaRs of benefit and contribution become large when $K^{(0)}$ is large.

The parameter $K^{(1)}$ is the annual amortization/decumulation ratio of deficiency/surplus assigned to a sponsoring company. When $K^{(1)} = 0$, a sponsoring company does not amortize/decumulate the deficiency/surplus, and therefore the contribution is fixed. When $K^{(1)} = 1$, the contribution is volatile because all of the deficiency/surplus assigned to a sponsoring company are amortized/decumulated. Therefore, as $K^{(1)}$ is large, the amortization is likely to change to the funding ratio. $K^{(1)}$ makes the same effect on the contribution as the parameter $K^{(0)}$ as shown in Figures 1. Therefore, the lower middle graph of Figure 8 are similar to the lower left graph for the contribution. On the other hand, $K^{(1)}$ does not make effect on the actual benefit directly. The actual benefit is a little bit affected indirectly by the distribution of the plan asset which depends on the amortization, and therefore it is less sensitive to $K^{(1)}$ than $K^{(0)}$. The result shows the CVaR of benefit is insensitive to $K^{(1)}$, and the mean of benefit is smaller for a larger $K^{(1)}$. The reason is as follows. When $K^{(1)}$ is large, the contribution becomes small for the large plan asset. This leads to the result that the plan asset becomes small, and the mean of benefit becomes small[7].

The parameter $K^{(2)}$ is the fraction of deficiency/surplus the retirees will share with active participants, and it also means the fraction assigned to the benefit instantaneously. It makes the same effect on the benefit as the contribution paid by a sponsoring company. The parameter $K^{(2)}$ makes the same effect on the benefit as $1 - K^{(0)}$ as shown in Figures 2. The lower right graph of Figure 8 are opposite to the lower left graph for the benefit. Therefore, the mean of benefit is large and the CVaR is small when $K^{(2)}$ is large. On the other hand, $K^{(2)}$ does not make an effect on the contribution. However, the result shows the mean of contribution becomes larger and the CVaR becomes smaller for a larger $K^{(2)}$. The reason is as follows. When $K^{(2)}$ is large, the benefit becomes large for the large plan asset. This leads to the result that the plan asset becomes small, and the mean of contribution becomes large. On the other hand, the benefit becomes small for the small plan asset when $K^{(2)}$ is large. This leads to the result that the plan asset becomes large, and the CVaR of contribution becomes small.

We show the relationship between three kinds of sharing parameters and mean benefits/contributions in Table 4.

[7] As well as $K^{(0)}$, the mean of benefit is larger for a larger $K^{(1)}$ in Case Ca.

Table 4: Relationship between sharing parameters and mean benefits/contributions

| | benefit | | contribution | | Who share the fraction? | |
|---|---|---|---|---|---|---|
| | mean | CVaR | mean | CVaR | $K^{(i)}$ | $1-K^{(i)}$ |
| $K^{(0)}$ | $-/+$ | $+$ | $-/+$ | $+$ | sponsor | participants/retirees |
| $K^{(1)}$ | $-/+$ | $+$ | $-/+$ | $+$ | sponsor | |
| $K^{(2)}$ | $+/-$ | $-$ | $+/-$ | $-$ | retiree | participants |

※ 'mean' is dependent on portfolio return: high return/low return

$K^{(0)}$ and $K^{(1)}$ are related with a sponsoring company, and they affect the CVaR of the contribution positively. On the other hand, $K^{(2)}$ is related with retirees, and it affects the CVaR of the benefit negatively. The signs of means of both benefit and contribution are dependent on portfolio return. We find the sensitivity to sharing parameters in the numerical analysis as in this table.

### 3.4. Sensitivity analysis of management fee

A management fee of the DC plan is 150bp(basis point), and those of other plans are 50bp. The fee affects the benefit and contribution. We conduct the sensitivity analysis of the four kinds of fee: $m = 0$, 50, 100, 150bp. We show the results in Figure 9.

The benefits of the DB and CB plans are not dependent on the fee, and the contribution of the DC plan is deterministic. Except those values, the benefits become small, and the contributions become large as the fee becomes large. The sensitivity of the mean to the fee is larger than that of the CVaR. We show the sensitivity coefficients per 100bp (1%) fee in Table 5 to compare those of four plans.

In the RS plan, the contribution is more sensitive to the fee than the benefit, and the average sensitivity of contribution is about eleven to twelve times as large as that of benefit. The

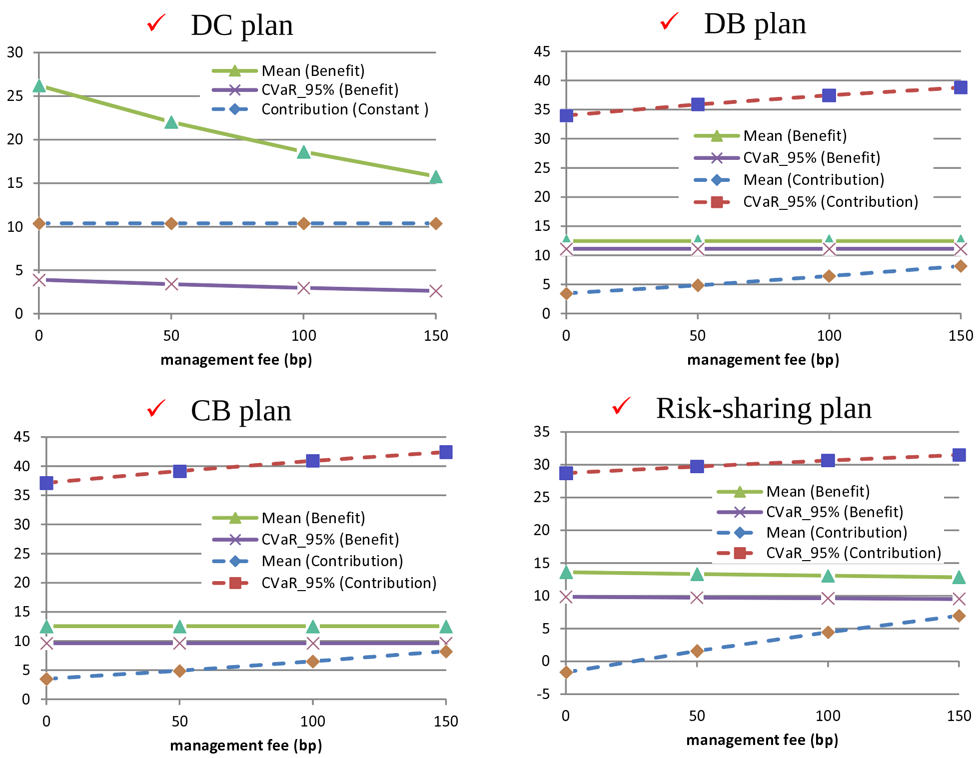


Figure 9: Sensitivity analysis of management fee for four plans

reason is as follows. The average coefficient of benefit to the deficiency/surplus of funding ratio

Table 5: Sensitivity of the benefits and contributions to the management fee

| | | Case (A) | | | | Case (B) | | | | Case (C) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| port | | benefit | | contribution | | benefit | | contribution | | benefit | | contribution | |
| folio | plan | mean | CVaR | mean | CVaR | mean | CVaR | mean | CVaR | mean | CVaR | mean | CVaR |
| [a] | DC | −6.95 | −0.85 | | | −5.61 | −0.73 | | | −2.19 | −0.33 | | |
| | DB | | | 3.14 | 3.19 | | | 3.99 | 2.79 | | | 2.99 | 1.57 |
| | CB | | | 3.17 | 3.53 | | | 4.01 | 3.18 | | | 3.06 | 1.82 |
| | RS | −0.51 | −0.22 | 5.74 | 1.85 | −0.45 | −0.22 | 5.21 | 1.79 | −0.25 | −0.15 | 2.78 | 1.12 |
| [b] | DC | −3.71 | −1.31 | | | −3.72 | −1.33 | | | −2.95 | −1.07 | | |
| | DB | | | 3.95 | 3.31 | | | 4.14 | 3.45 | | | 3.40 | 2.69 |
| | CB | | | 4.04 | 3.69 | | | 4.21 | 3.80 | | | 3.51 | 3.03 |
| | RS | −0.34 | −0.23 | 3.85 | 2.51 | −0.34 | −0.23 | 3.94 | 2.63 | −0.29 | −0.21 | 3.24 | 2.14 |

in Case Aa is 0.00895 in the base analysis. [8] The coefficient of contribution of funding ratio is $K^{(0)}K^{(1)} = 0.1$, and therefore it is about eleven times as large as the coefficient of benefit. On the other hand, the sensitivity ratio in Case Aa, calculated in Table 5, is 11.2, and therefore we confirm it coincides with the value as stated above. According to Table 5, we find the sensitivity is dependent on the coefficient to the deficiency/surplus of funding ratio which is affected by the parameters.

Next, we compare the sensitivity of the RS plan to other plans. The mean benefit of the DC plan is about ten times as sensitive as that of the RS plan, and the CVaR of the benefit is about five times. We find the DC plan is much sensitive to the fee. The CVaRs of contributions of the DB and CB plans are about twice as sensitive as that of RS plan. The sensitivity of the DB plan is slightly smaller than that of the CB plan. On the other hand, the contribution of the RS plan is more sensitive to those of DB and CB plans in the case where the interest profit is obtained from investment return such as Case Aa. This reason is that the decumulation from the plan asset is affected by the management fee.

## 4. Backtesting

We implement the backtest in order to examine the actual impact on the pension plans. We compare four plans using the historical data of twenty years.

### 4.1. Setting

The data and parameters for backtesting are as follows.

- Backtest period: From March 1995 to March 2015 (twenty years)
- Historical data
  - — Domestic stock (DS): TOPIX (Tokyo Stock Price Index)
  - — Domestic bond (DB): JPGBI (Citigroup Japan Government Bond Index)
  - — Foreign stock (FS): S&P500 (Standard & Poor's 500 Stock Index)
  - — Foreign bond (FB): USGBI (Citigroup USA Government Bond Index)
  - — Wage growth rates, which are calculated using monthly labor survey (Seasonally adjusted wage indices, total cash earnings, establishments with thirty employees or more)
  - — 10-year government bond yield, which are available from a website of interest rate of government bond, Ministry of Finance
  - — Dollar-yen exchange rate (center value of interbank spot rate)

[8] The coefficient of benefit to the deficiency/surplus of funding ratio is $\left(1 - K^{(0)}\right) K^{(2)} \left(\frac{B_n^{(s)}}{L_n^{(s)}}\right)$.

— Japanese yen interest rate: one-year Euroyen TIBOR
— Dollar interest rate: one-year Eurodollar interest rate

Table 6: Statistics (local currency basis)

| | DS | DB | FS | FB | IR | 10Y-GB |
|---|---|---|---|---|---|---|
| index | TOPIX | JPGBI | S&P500 | USGBI | wage growth rate | bond yield |
| mean | 3.87% | 2.73% | 12.07% | 8.01% | −0.36% | 1.59% |
| st. dev. | 25.71% | 2.37% | 25.73% | 12.86% | 1.72% | 0.80% |

- Parameters used in calculating initial actuarial liabilities
  - — annual mean government bond yield from March 1990 to March 1994 (nominal: 5.52%, real: 2.94%)
  - — annual mean wage growth rate from March 1990 to March 1994 (2.53%)
- Parameters for risk-sharing plan: $T^{(1)} = 1.05$, $T^{(2)} = 1.3$, $K^{(0)} = 0.5$, $K^{(1)} = 0.2$, $K^{(2)} = 0.5$ (which are the same as the parameters in Section 3.1.)
- Currency and hedging strategies: The results are evaluated on a yen basis. Two kinds of foreign exchange hedging strategies are evaluated; no hedge and perfect hedge [9]
- Portfolio: Constant rebalance strategy with the following three kinds of weights (rebalance at the end of every March)

| | DS | DB | FS | FB | Stock:Bond |
|---|---|---|---|---|---|
| Portfolio [a] | 25% | 35% | 25% | 15% | 5:5 |
| Portfolio [b] | 5% | 55% | 5% | 35% | 1:9 |
| Portfolio [c] | 50% | 0% | 50% | 0% | 10:0 |

- Management fees: 150bp for the DC plan, and 50bp for other plans (which are the same as the parameters in Section 3.1)

### 4.2. Results

We show the cumulative return in Figure 10 when we invest four assets using the constant rebalance strategy. The left graph shows the real cumulative return with no hedging, the middle graph shows them with perfect hedging, and the right graph shows the mean and standard deviation of the real rate of return. Portfolio [b] which bond weight is the largest has the lowest risk and return,

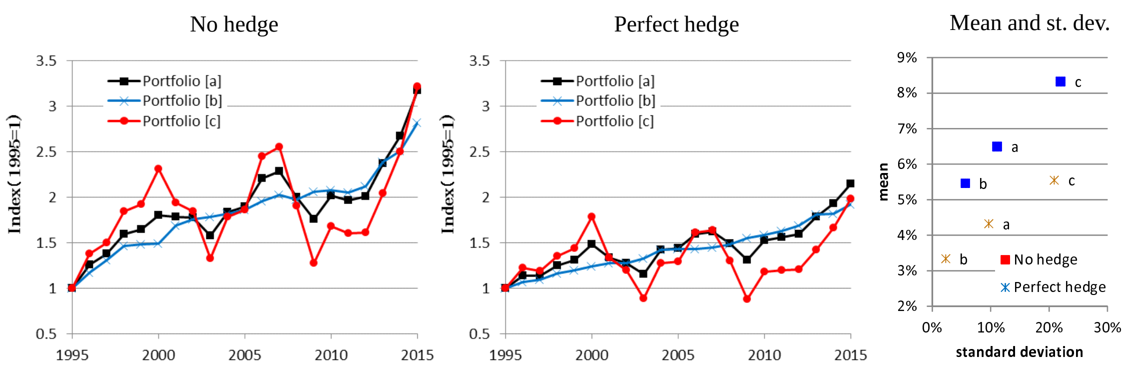


Figure 10: Cumulative return

[9]The perfect hedge is implemented using the theoretical future price based on the difference of interest rates between Japanese yen and US dollar.

whereas portfolio [c] which stock weight is the largest has the highest risk and return. We find we get the largest return actually by investing the high-risk portfolio. Moreover, no hedging is a riskier strategy than the perfect hedging, but the actual standard deviations of no hedging strategy are almost the same as those of the perfect hedging. However, the actual mean returns of no hedging strategy are larger than those of perfect hedging.

Due to space limitation, we show the results of no hedging strategy hereafter. We observe the outcomes of benefits and contributions of four plans for twenty years. We show the benefits and contributions of three kinds of portfolios with no hedging strategy in Figure 11 and Figure 12. [10]

At first, we examine the benefits in Figure 11. The benefits of the DB and CB plans in the three graphs are the same outcomes, respectively, because they are independent of the portfolios.

The benefits of the DB plan are stably increasing, whereas those of the CB plan are stably decreasing. As the result, the benefits of the DB plan are larger than those of the CB plan. The reason is as follows. The reason that the benefits of the DB plan are increasing is that the backtest period is almost in deflation. The value of benefit of DB plan increases due to the deflation and the fixed nominal value. The reason that the benefits of the CB plan are decreasing is that the guaranteed rate used to set the pay credit is far apart from the 10-year government bond yield. The real interest rate used to calculate the normal contribution (pay credit) 2.94%, but the average rate in the backtest period is $1.95\%(=1.59\%-(0.36\%))$.The difference between them is about 1%, and this makes it difficult to keep the actuarial liability, and pay the benefit. This is the problem to solve in the CB plan which is difficult to change the pay credit rate.

The descending orders of the mean and standard deviation of cumulative return are portfolio [c], [a], [b] as shown in the right-hand side of Figure 10. This is reflected to the benefits of the DC and RS plans. The variability of benefits is similar to the cumulative return in the backtest period, however the variability of the DC plan is larger than that of the RS plan. Next, we

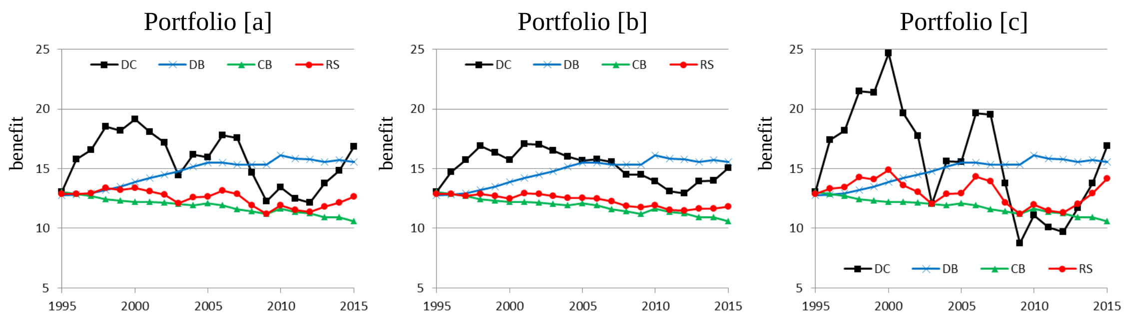


Figure 11: Benefits derived by backtesting

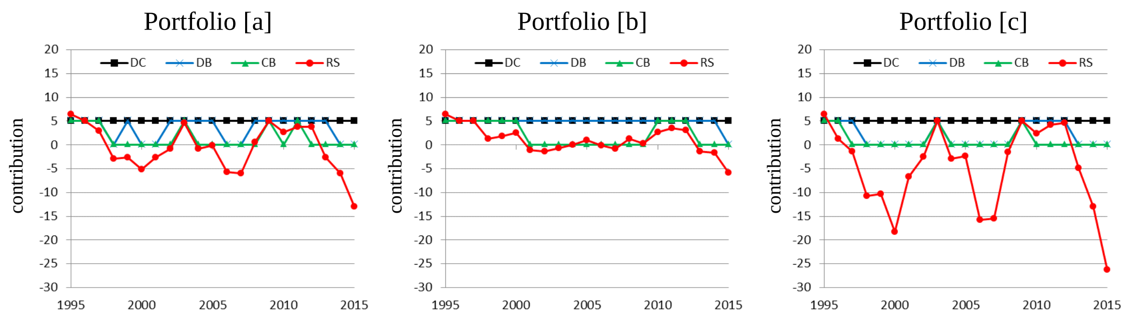


Figure 12: Contributions derived by backtesting

[10]The benefits with no hedging are larger than those of perfect hedging, and The contributions with no hedging are smaller than those of perfect hedging.

examine the contributions in Figure 12. The contribution of the DC plan is deterministic. The maximum contributions of other plans are the same values for three portfolios with no hedging strategy. The reason is that the surpluses are not below zero, and we do not need the amortizations for all portfolios. The contributions of the DB and CB plans are zero in almost years because the normal contributions are zero when the funding ratio is beyond 150%. The contributions of the RS plan become negative in many years because the amortizations are negative when the funding ratio is beyond 130%. We can check the reason by examining the funding ratio in Figure 13.

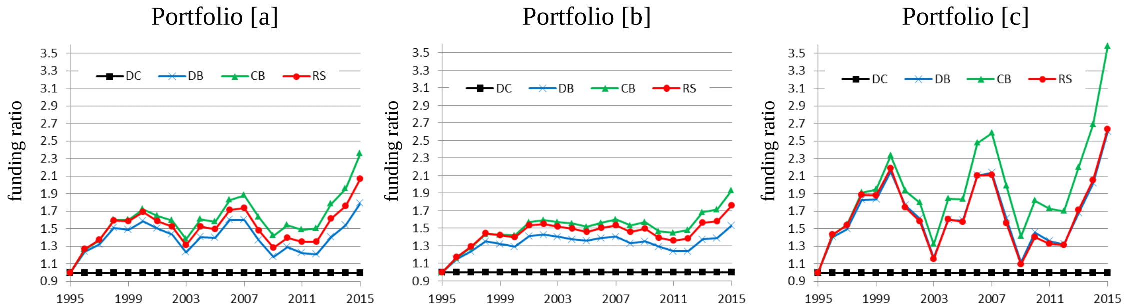


Figure 13: Backtest: funding ratio

We examine the mean and CVaR diagrams of benefits and contributions derived using no hedging strategy in Figure 14. The benefit becomes better toward upper right side in the diagram, and the contribution becomes better toward lower left side. The benefits of the DB and CB plans are not dependent on portfolios.

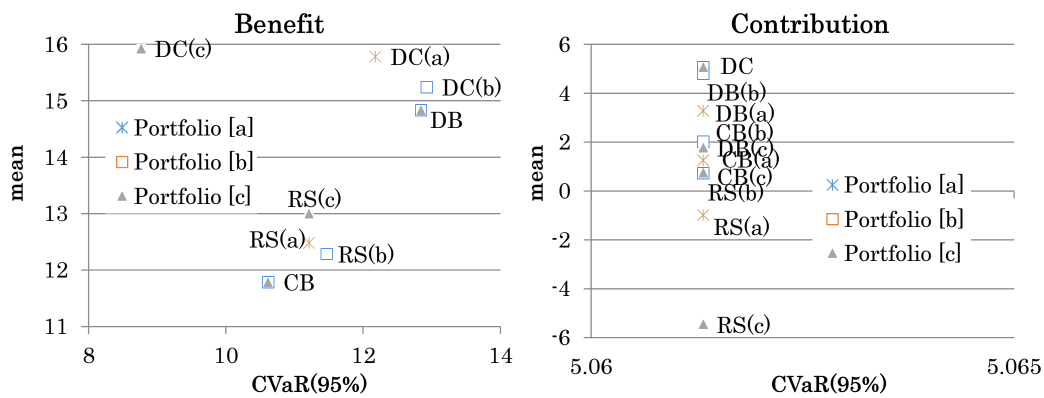


Figure 14: Mean and CVaR of benefits and contributions derived using no hedging strategy

The relationship of the mean benefit among four plans are DC > DB > RS > CB. The cumulative return is 2.5 to 3 times the initial value regardless of the portfolios, and therefore the market condition in the backtest period is favorable for the DC plan. The relationship of CVaRs of benefits is DC(b) > DB > DC(a) > RS > CB > DC(c), and it is DB > RS > CB except the DC plan as well as that of the mean. The CVaR of the DC plan differs depending on portfolios, and the CVaR becomes large as portfolio risk is low.

The relationship of mean contribution is RS < CB < DB < DC for each portfolio. The contribution of the DC plan which is deterministic is larger than those of other plans. Especially, the contribution of the RS plan is the smallest because it can be negative for the funding ratio beyond the threshold. On the other hand, the CVaRs of contributions are the same for four plans. The reason is that the surpluses are not below zero, and we do not need the amortizations as stated above. Therefore, the normal contribution is equal to the maximum contribution which becomes

95% CVaR for twenty samples [11].

## 5. Conclusion

In this paper, we design the risk-sharing pension plan using the five parameters which control the level of risk sharing. We implement the Monte Carlo simulation for a long-term period, and evaluate the uncertainty of benefits and contributions. We can offer some suggestions about the parameters. Moreover, we compare the RS plan with the existing DC, DB, and CB plans, and we find the benefit and contribution of the RS plan are not only between the DB and DC plans, but also superior to them in some cases. In the future research, we compare the risk-sharing plan proposed in this paper with the intermediate plan which consists of the weighted plan of the DB and DC plans. In addition, we need to formulate the optimization model, which solve the problem to find the optimal parameter values of controlling the risk-sharing.

### Acknowledgment

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## Appendix

### A. Mathematical expressions for three existing plans

#### A.1. Normal contribution rate

We calculate the normal contribution rate required to secure benefits in the specified DB plan, and apply it to CB and DC plans in order to match the initial actuarial liabilities among all pension plans. Therefore, we utilize the real interest rate as the guaranteed interest rate for the DB plan. The normal contribution rate $p^1$ is calculated by Equation (A.1).

$$p^1 = \frac{\displaystyle\sum_{k=T_L}^{T_L+T_R-1} \exp\left\{-(k-T_L)\overline{J}'\right\}}{\displaystyle\sum_{k=0}^{T_L-1} \exp\left\{(T_L-k)\overline{J}\right\}} \tag{A.1}$$

#### A.2. Actuarial liability and benefit for each period and each age

The calculating formulas of actuarial liability and benefit are dependent on each period and each age. We express the formulas of actuarial liability and those of pension benefit for retirees.

**(1) DC plan and CB plan**

The actuarial liability and benefit are commonly described as follows. However, we calculate the initial actuarial liabilities of DC and CB plans using the real government bond yield so that those are the same values in order to compare with one another in Section 3.

$$y_n^{(s)} = \begin{cases} r_{M,n}^{(s)} - m & \text{(DC plan)} \\ j_n^{(s)} & \text{(CB plan)} \end{cases}$$

The actuarial liabilities and benefits are calculated as follows. [12]

**A. Actuarial liability**

① Actuarial liability of participants ($x = 1, \ldots, T_L - 1$)

$$L_{x,n}^{(s)} = \begin{cases} p^1 \displaystyle\sum_{k=0}^{x-1} \exp\left\{(x-k)\overline{J}\right\} & (n=1) \\ \left(L_{x-1,n-1}^{(s)} + p^1\right) \exp\left(y_{n-1}^{(s)}\right) & (n = 2, \ldots, N) \end{cases}, \quad \text{where } L_{0,n}^{(s)} = 0 \tag{A.2}$$

② Actuarial liability of initial retires ($x = T_L, \ldots, T_L + T_R - 1$)

$$L_{x,n}^{(s)} = \begin{cases} p^1 \displaystyle\sum_{k=0}^{T_L-1} \exp\left\{(x-k)\overline{J}\right\} & (n = 1 \text{ and } x = T_L) \\ \left(1 - \frac{x-T_L}{T_R}\right) \cdot A^{(s)} \cdot \exp\left((x-T_L)\overline{J}\right) & (n = 1 \text{ and } x \neq T_L) \\ \left(1 - \frac{x-T_L}{T_R}\right) A^{(s)} \exp\left\{(x - T_L + 1 - n)\overline{J} + \displaystyle\sum_{l=1}^{n-1} y_l^{(s)}\right\} & (n = 2, \ldots, \max(x - T_L + 1, 2)) \end{cases} \tag{A.3}$$

where $A^{(s)} = L_{T_L,1}^{(s)}$

③ Actuarial liability except for initial retirees ($n = 2, \ldots, N$)

$$L_{T_L+k,n+k}^{(s)} = \begin{cases} \left(L_{T_L-1,n-1}^{(s)} + p^1\right) \exp\left(y_{n-1}^{(s)}\right) & (k=0) \\ \left(1 - \frac{k}{T_R}\right) \cdot L_{T_L,n}^{(s)} \cdot \exp\left\{\sum_{l=n}^{n+k-1} y_l^{(s)}\right\} & (k = 1, \ldots, \min(T_R - 1, N - n)) \end{cases} \tag{A.4}$$

[12] When $n = 1$, the values are independent of $s$, but we put '$(s)$' for calculation at time $n \geq 2$

**B. Benefit** ($x = T_L, \ldots, T_L + T_R - 1$, $n = 1, \ldots, N$)

$$B_{x,n}^{(s)} = \frac{L_{x,n}^{(s)}}{T_L + T_R - x} \tag{A.5}$$

**(2) Specific DB plan**

We use the advanced funding method where the funding standard is calculated using a real government bond yield at the evaluation time. The real actuarial liabilities of participants are the same as those of CB plan. The real benefit of the beginning of qualification period is equal to 1, and we fix it at a nominal value. This means that the real value decreases by an inflation. The actuarial liabilities of retirees are calculated based on the benefit.

**A. Real actuarial liability**

① Real actuarial liability of participants[13] ($n = 1 \ldots, N$, $x = 1, \ldots, T_L - 1$)

$$L_{x,n}^{(s)} = p^1 \sum_{k=0}^{x-1} \exp\left\{(x-k)\overline{J}\right\}, \text{ and } L_{0,n}^{(s)} = 0 \tag{A.6}$$

② Real actuarial liability of retiree ($n = 1 \ldots, N$, $x = T_L, \ldots, T_L + T_R - 1$)

The real actuarial liability of retiree is calculated based on the benefit shown by Equation (A.8) below.

$$L_{x,n}^{(s)} = \left\{ \sum_{l=0}^{T_L+T_R-x-1} \exp(-l\overline{J}') \right\} B_{x,n}^{(s)} \tag{A.7}$$

**B. Real benefit**

① Real benefit for initial retirees ($x = T_L, \ldots, T_L + T_R - 1$)

$$B_{x,n}^{(s)} = \begin{cases} \exp\left\{-(x - T_L)\overline{I}\right\} & (n = 1) \\ B_{x,n-1}^{(s)} \exp\left\{-\left(i_{n-1}^{(s)} - \overline{I}\right)\right\} & (n = 2, \ldots, \max(x - T_L + 1, 2)) \end{cases} \tag{A.8}$$

② Real benefit except for initial retirees ($n = 2, \ldots, N$)

The real benefit in period $n$ for retirees who reach the age of $T_L$ after the second period is

$$B_{T_L,n} = 1,$$

and the benefit after $n + 1$th period is calculated as follows because we assume that it decreases by an inflation.

$$B_{T_L+k,n+k}^{(s)} = \exp\left(-\sum_{l=0}^{k-1} i_{n+l}^{(s)}\right) \quad (k = 1, \ldots, \min(T_R - 1, N - n)) \tag{A.9}$$

### A.3. Actuarial liability, benefit, contribution and plan asset for each period

**(1) Actuarial liability and benefit**

The actuarial liability and benefit calculated for each age are aggregated respectively for each period as follows.

$$\text{benefit} \ : \ B_n^{(s)} = \sum_{x=T_L}^{T_L+T_R-1} B_{x,n}^{(s)} \tag{A.10}$$

$$\text{actuarial liability} \ : \ L_n^{(s)} = \sum_{x=0}^{T_L+T_R-1} L_{x,n}^{(s)} = L_n^{a(s)} + L_n^{p(s)} \tag{A.11}$$

$$\text{where } L_n^{a(s)} = \sum_{x=0}^{T_L-1} L_{x,n}^{(s)}, \ L_n^{p(s)} = \sum_{x=T_L}^{T_L+T_R-1} L_{x,n}^{(s)} \tag{A.12}$$

[13] It is the same as the liability of CB plan

**(2) Contribution and plan asset**

The contribution consists of the normal contribution and amortization. The normal contribution is constant, and the amortization is zero in the DC plan. We adopt the rule in the DB and CB plans that we do not pay the normal contribution in the period when the funding ratio is beyond 150%. On the other hand, the amortization of funding deficiency is paid when the funding ratio is under one, and the amortization rate denotes $p^{2(s)}$. The contribution and plan asset are calculated as follows.

○ **DC plan**

$$\begin{aligned} \text{normal contribution} \ &: \ C_n^{1(s)} = \sum_{x=0}^{T_L-1} p^1 = T_L \cdot p^1 \\ \text{amortization} \ &: \ C_n^{2(s)} = 0 \ (p^{2(s)} = 0) \\ \text{plan asset} \ &: \ F_n^{(s)} = L_n^{(s)} \end{aligned}$$

○ **DB plan and CB plan**

The deficiency/surplus in period $n$ is defined as the difference of plan asset and actuarial liability in the following.

$$S_n^{(s)} = F_n^{(s)} - L_n^{(s)}$$

The normal contribution is dependent on the funding ratio, and the amortization is dependent on the surplus. They are calculated as follows,

$$\text{normal contribution} \ : \ C_n^{1(s)} = \begin{cases} T_L \cdot p^1 & (F_n^{(s)} < \theta L_n^{(s)}) \\ 0 & (F_n^{(s)} \geq \theta L_n^{(s)}) \end{cases},$$

$$\text{amortization} \ : \ C_n^{2(s)} = \sum_{x=0}^{T_L-1} p_n^{2(s)} = T_L \cdot p_n^{2(s)} = \begin{cases} -S_n^{(s)} K^{(1)} & (S_n^{(s)} < 0) \\ 0 & (S_n^{(s)} \geq 0) \end{cases},$$

$$\text{contribution} \ : \ C_n^{(s)} = C_n^{1(s)} + C_n^{2(s)} = \begin{cases} T_L \cdot p^1 + \left(L_n^{(s)} - F_n^{(s)}\right) K^{(1)} & (F_n^{(s)} < L_n^{(s)}) \\ T_L \cdot p^1 & (L_n^{(s)} \leq F_n^{(s)} < \theta L_n^{(s)}) \\ 0 & (F_n^{(s)} \geq \theta L_n^{(s)}) \end{cases},$$

$$\text{plan asset} \ : \ F_n^{(s)} = \begin{cases} L_1^{(s)} \cdot f & (n = 1) \\ \left(F_{n-1}^{(s)} + C_{n-1}^{(s)} - B_{n-1}^{(s)}\right) \exp\left(r_{M,n-1}^{(s)} - m\right) & (n \geq 2) \end{cases}$$

where $\theta$ is the upper funding boundary ($\theta = 1.5$), $K^{(1)}$ is a constant fraction of funding deficiency, and $f$ is a constant initial funding ratio. We show the relationship between contributions and funding ratio for the DB and CB plans in Figure 15.

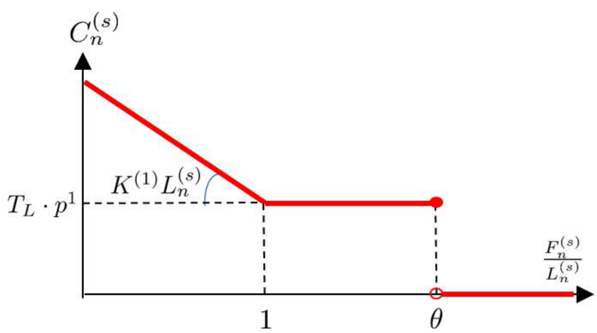


Figure 15: The relationship between contributions and funding ratio for DB and CB plans