Estimating the Recovery Rates for Unsecured Loans to Small Sized Firms

Masahiro Toshiro[1], Masao Tasaki[2], Yusuke Hikidera[3], Norio Hibiki[4]

[1,2,3] Micro Business and Individual Unit, Japan Finance Corporation*

[4] Faculty of Science and Technology, Keio University

Abstract

We analyze the recovery rates of 66,928 Japanese unsecured loans in default by ordered logistic regression. We divide the defaulted firms by sole proprietorships and industrial corporations and analyze the recovery rates for each type of firms. The recovery rate for sole proprietorships is larger than that for industrial corporations. Moreover, we model not only the recovery rate during five years at the time of default but also that evaluated at the time of loan appraisal for each type of firms, and we call them "loan model" and "after-default model" respectively. The significant factors with large regression coefficients are different for each model and each type of firms. We find that these are (1) guarantee by business owner's family in two models for each type of firms, (2) firm age in two models for industrial corporations, (3) exposure rate at default in the after-default model for each type of firms, (4) obligor's real-estate value minus debt amount, initial loan amount, and white tax return in the loan model for sole proprietorships. The values of Somers' D for the after-default model is larger than those for the loan model because the exposure rate at default which has large estimates can be available at time of default. The value of Somers' D for sole proprietorships is larger than that for industrial corporations. We divide all defaulted loans into four classes based on the score evaluated by the model, and validate the ratings of the actual recovery rates through three kinds of statistical tests. In addition, we conduct out-of-sample tests, and examine the usefulness of the model.

Keyword: credit risk, recovery rates, probability of default, unsecured loan, small sized firms

1. Introduction

As stated in the Basel Capital Accord regulating capital adequacy ratios of financial institutions(Basel Committee on Banking Supervision (2011)), the expected loss (EL) of the loan is calculated by multiplying the exposure at default (EAD) by the probability of default (PD) and the loss given default (LGD). The LGD is obtained as one minus recovery rate (RR). Therefore, the estimation of RR after default is important in evaluating credit risk of loans as well as the estimation of PD. There are a lot of studies concerning the PD (Duffie and Singleton (2003), Bluhm, Overbeck and Wagner (2010), Shirata (2003), Moridaira (2009), Yamashita and Miura (2011)), and the financial institutions employ the PD estimation models. On the other hand, there are a few studies concerning the RR. In particular, there are few studies concerning the statistical models based on the data of actual collections from defaulted loans, not based on market data and risk premiums. The use of RR models has not been established except for a few financial institutions. There are two internal rating based approaches (IRB) of the Basel Capital; fundamental internal rating based

* Any views or opinions expressed in this paper are solely those of the authors and do not necessarily represent those of Micro Business and Individual Unit of Japan Finance Corporation.

approach (FIRB) and advanced internal rating based approach (AIRB). When the FIRB is adopted, the estimation of PD is essential, but that of LGD is not required because the population for the PD estimation is all obligors of loans, but the population for the RR estimation is limited to defaulted loans, and it is difficult to obtain enough number of observation for modeling. Furthermore, it takes a long term to recover the money from the obligors of defaulted loans, which also makes the estimation difficult.

There are some previous studies of the statistical models, using the data of actual recoveries from the defaulted loans to small and medium-sized enterprises (SMEs). Dermine and Neto de Carvalho (2006) model recovery rates by fractional response regressions using 374 defaulted loans which Banco Comercial Português (BCP), private bank in Portugal, has financed SMEs. Bastos (2010) also develops two alternative models in forecasting recovery rates using 374 defaulted loans (average EAD: 140 thousand euros) to SMEs by BCP; fractional response regression model and a nonparametric and nonlinear regression tree model. Gurtler and Hibbeln (2011) identify relevant pitfalls in modeling workout LGD, and propose the methods to avoid these problems on a data set of 71,463 defaulted loans (average EAD: nine thousand euros) financed by a German bank. However, most loans are secured, and only 2,575 unsecured loans are included.

There are also some previous studies for Japanese banks. Itoh and Yamashita (2008) built the RR estimation models using 2,603 actual recoveries in subrogation repayment by the three Japanese credit guarantee associations. They develop the binomial logistic and ordered logistic regression models with several factors. Kawada and Yamashita (2013) construct the PD estimation models using 867,885 loans of three banks in Japan (average loan balance: 110 million yen), and they develop the RR estimation models using 6,718 defaulted loans with the factors at the default. Tanoue, Kawada and Yamashita (2017) construct the PD estimation models using 679,607 loans of three banks in Japan (average loan balance of 110 million yen) and they develop the multi-stage LGD estimation models using 8,732 defaulted loans with the similar factors to Kawada and Yamashita (2013). Moreover, they propose the EL estimation model which consists of the PD and multi-stage LGD estimation.

As described above, many different types of models have been proposed, but they have been built with the aim of evaluating credit risk of loan balance, and there are no models using factors at the time of loan appraisal. Then, Ogi, Toshiro and Hibiki (2015) analyze the RR using 11,689 loans in default to small sized firms, and they construct the three ordered logistic regression models with factors on the loan appraisal for the RR estimation with respect to each degree of real estate collateral coverage: fully secured loan model, partially secured loan model, and unsecured loan model. To the best of our knowledge, there are no RR estimation models which are constructed involving the degree of real estate collateral coverage or the existence of guarantees. The unsecured loan model was built using 6,650 defaults with financial status, firm age of obligor and initial loan amount. In practice, when we estimate credit risk of real estate secured loans and guaranteed loans, it is important to calculate the coverage rate precisely, based on the real estate collateral value or the existence of guarantee, and therefore we do not have a strong need for refined modeling. The Strategic Directions and Priorities formulated by the Japanese Financial Services Agency (2015b) in September 2015 stipulates from the 2015 business year that "the JFSA will encourage financial institutions to actively contribute to the creation of customer corporate values, the sustainable growth of the national economy, and the revitalization of local economies by supporting customers' efforts to enhance their business models and by underwriting loans relying on customers' future business prospects, not just on collateral and guarantees.". This shows that the need for the RR estimation model after default on unsecured and unguaranteed loans is increasing, because the number of loans not depending on collateral and guarantees is expected to increase in the future.

This paper is extended, based on the work of Ogi et al. (2015) which analyze the factors affecting the RR and develop the RR model using the data of 6,650 loans in default unsecured and unguaranteed by third-party to small sized firms made by Micro Business and Individual Unit of Japan Finance Corporation (JFC-Micro). We analyze the RR using the data of 66,928 loans in default which is more than ten times as much as Ogi et al. (2015). In addition, we use the monthly observed recoveries, whereas Ogi et al. (2015) observed the recoveries annually. This makes it possible to observe the immediate recoveries after default, even within the same year of default. Furthermore, while Ogi et al. (2015) propose a model as a support tool for evaluating new loans, we also build a model as a risk management tool for improving the RR after default. We develop two types of RR models for loans to sole proprietorships and industrial corporations respectively.

The purpose and contribution of our study are as follows. First, we analyze the recovery rates of 66,928 Japanese unsecured loans in default by ordered logistic regression. We divide the defaulted firms by sole proprietorships and industrial corporations, and analyze the recovery rates for each type of firms. The recovery rate for sole proprietorships is larger than that for industrial corporations. We model not only the recovery rate during five years at the time of default but also that evaluated at the time of loan appraisal for each type of firms, and find the significant factors that affect the RR after default. The guarantee by business owner's family is statistically significant in two models for loans to each type of firms. The firm age is significant for the models, especially for loans to industrial corporations. The significant factors at the time of default are exposure rate at default (EAD rate) for loans to each type of firms. The significant factors at the time of providing new loans are obligor's real-estate value minus debt amount, initial loan amount, and white tax return for loans to sole proprietorships. Second, we examine two types of RR models for loans to sole proprietorships and industrial corporations. We give the ratings to the loans based on the score estimated from the RR model. Four ratings are given to the loans based on the score estimated from the RR model. We evaluate the order of the actual RRs by rating, and examine the usefulness in practice. We find the appropriate result that the higher the score is, the higher the actual RR is in each type of models for each type of firms. Furthermore, we calculate the actual RR by each rating and fiscal year of default and providing loan in order to evaluate the robustness in time series. We find that the actual RR of each rating is in a proper order for each fiscal year of default and providing loan.

Our paper is organized as follows. Section 2 gives results of basic data analysis for RR. We show the modelling method in Section 3, and the estimation results in Section 4. We evaluate the models in Section 5, and conduct out-of-sample tests in Section 6. Section 7 provides our concluding remarks and future research.

2. Basic data analysis for recovery rates

2.1 Data

This study uses in-sample data of 66,928 loans in default from fiscal year (FY) 2008 to FY2011, which are unsecured and unguaranteed by third parties other than the business owners by JFC-Micro. These consist of 29,772 loans to sole proprietorships and 37,156 loans to industrial corporations. We also employ 14,624 loans defaulted in FY2012 as out-of-sample data, which consist of 6,305 loans to sole proprietorships and 8,319 loans to industrial corporations.

According to Japan Finance Corporation (2017), JFC was established on October 1 in 2008, as a policy-based financial institution by the Japanese government, which has owned 100% of JFC's stocks. It consists of three business headquarters: Micro Business and Individual Unit, Agriculture, Forestry, Fisheries and Food Business Unit, and Small and Medium Enterprise (SME) Unit. JFC-Micro provides business loans to small sized firms, start-up firms and reconstruction firms. It assumes a role of providing community and safety-net loans. It also provides educational loans which newly reach 120 thousand individuals every year. The summary of business loans in FY2017 is as follows. The number of obligors reaches 0.88 million small sized firms. The average loan balance per firm is 6.98 million yen, most of which are classified into small loans. Half of obligors are sole proprietorships, and approximately 90% of obligors are businesses with nine or fewer employees. Therefore, the loan amount is considerably small compared with the previous studies so far.

Table 1 shows the basic statistics of exposure at default (EAD).[1] The average EAD is 1.92 million yen for loans to sole proprietorships and 4.61 million yen for loans to industrial corporations. It is much smaller than other business finance loans in Japan.

Table 1: Basic statistics of exposure at default (EAD)

(unit: a million yen)

	# of loans	Mean	Std. dev.	Percentile				
				P10	P25	P50	P75	P90
Sole proprietorships	29,772	1.92	1.79	0.30	0.70	1.45	2.60	4.14
Industrial corporations	37,156	4.61	4.12	0.77	1.71	3.38	6.24	10.20

The loan conditions are shown in Table 2. Guarantee conditions are no guarantee or guarantee only by the business owner or his family, and there is no guarantee of any credit guarantee association, financial institution or third parties. Therefore, recoveries after default are limited to repayment from the obligors, or business owner and his family. In addition, the money has been recovered over several years after default.

Table 2: Loan conditions

Form	Collateral	Guarantee	Repayment
Loan on deed	Unsecured	No guarantee or guarantee only by business owner or his family	Monthly installment repayment

In our paper, obligors with more than three-month overdue loans, effectively bankrupt obligors and bankrupt obligors are defined as defaulted obligors. A bankrupt obligor is an obligor who faces a legal and formal bankruptcy. For example, it consists of bankruptcy, liquidation, corporate restructuring, corporate reorganization, civil rehabilitation, suspension of transactions at the clearing house. An effectively bankrupt obligor is an obligor who faces serious business difficulties, even though he does not face a legal and formal bankruptcy, and has no prospect of launching into reconstruction.

2.2 Determining the collection period

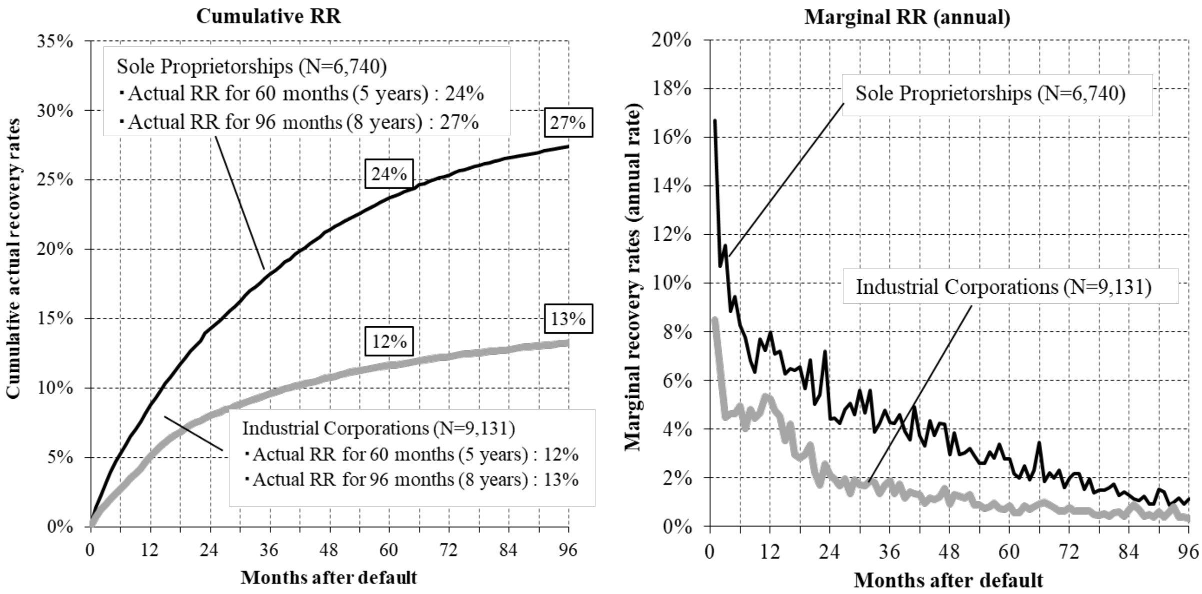
The recovery rate is dependent on the collection period, and therefore we examine the relationship between them. The recovery rate RR_{it} of the individual loan i is defined as the ratio of

[1] We define the EAD as principal exposure at default. The bank can require the obligors both the residual principal and the residual loan interest rates at the time of default by forfeiture of benefit of time. However, it is difficult to collect them from the defaulted obligors in practice. Therefore, banks prioritize to collect the principal exposure and exempt the obligors from a part of the residual loan interest rates. In our paper, we model the recovery in accordance with an actual practice, but it is our future task to consider the residual loan interest rates.

the cumulative amount of principal repayment over t months after default (C_{it}) to the exposure at default (EAD_i), where N is the number of individual loan observations.

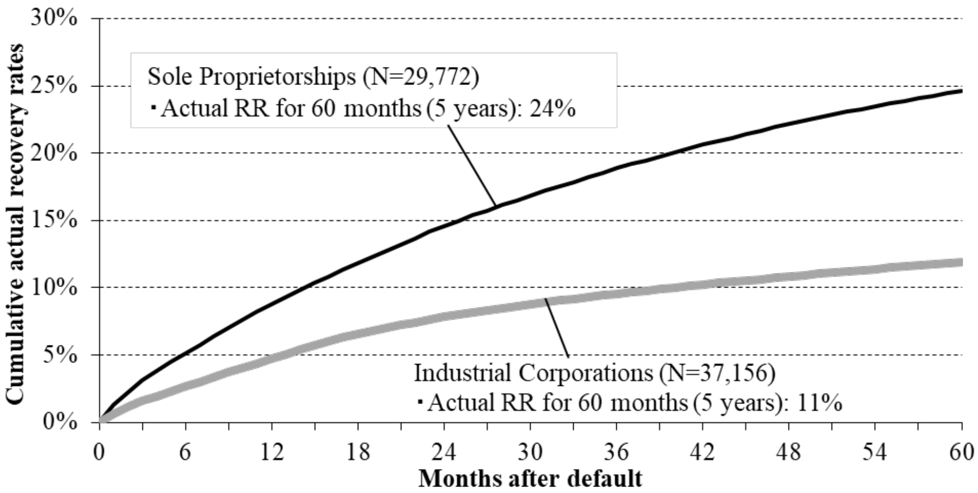
$$RR_{it} = \frac{C_{it}}{EAD_i} \ (i = 1, \cdots, N)$$

The left-hand side of Figure 1 shows the cumulative actual RR of the loans defaulted in FY 2008 so that we can observe the recovery data from defaulted loans as much as possible. As of the end of March 2017, we observe the data for 96 months after default. The cumulative actual RR for 96 months (eight years) after default is 27% for sole proprietorships and 13% for industrial corporations. The actual RR for sixty months (five years) after default is 24% for sole proprietorships and 12% for industrial corporations. The cumulative actual RR increases at a decreasing rate, especially after sixty months. The right-hand side of Figure 1 shows the marginal RR, which is the rate of monthly amount of recovery to the loan balance at the end of the previous month. The value is annualized by multiplying twelve. Looking at the marginal recovery rate by periods after default, it becomes less than 2% when it exceeds sixty months. We analyze the cumulative actual RR up to sixty months ($t = 60$) after default. We cannot use the data defaulted in five years from FY2013 to FY2017. Then, we use 66,928 loans in default from FY2008 to FY2011 as in-sample data for estimating the RR model, and we employ 14,624 loans defaulted in FY2012 data as out-of-sample data.



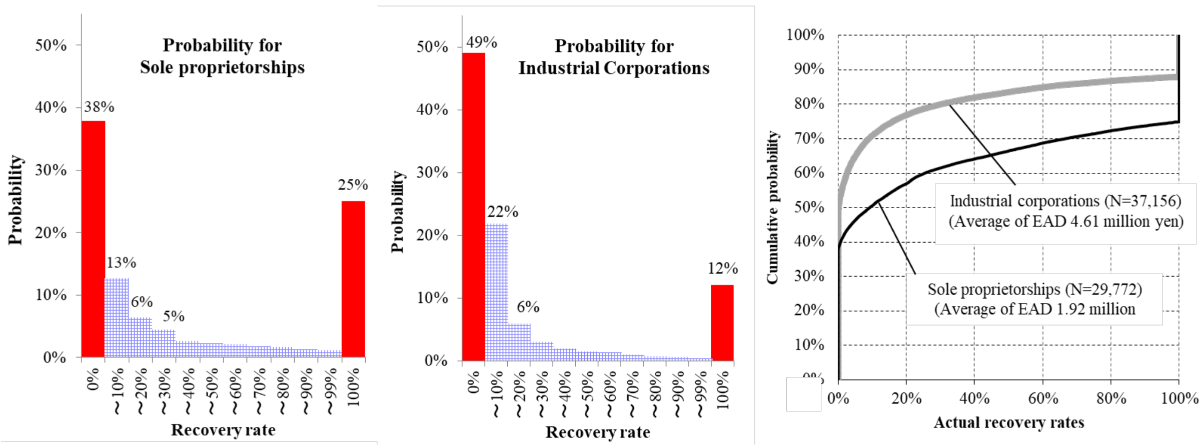
RRs of the loans defaulted in FY 2008

Figure 2 shows the cumulative actual RR up to sixty months after default of 66,928 loans defaulted from FY2008 to FY2011. The recoveries are gradually progressing because the payers are limited to the obligor, business owner or his family,



ctual recovery rates of defaulted loans from FY2008 to FY2011

Figure 3 shows the probability distributions of the mean actual RR for sixty months after default for sole proprietorships on the left-hand side of Figure 3, industrial corporations on the middle, and cumulative distribution on the right-hand side. We find the percentages of no recovery (0% RR) and full recovery (100% RR) are extremely high. No recovery accounts for 38% in loans to sole proprietorships, and full recovery accounts for 25%. As shown in Figures 1 to 3, the RR after default is higher for loans to sole proprietorships than industrial corporations.



3. Modelling the recovery rates

3.1 Methodology

As shown in previous studies, the characteristics of recovery rates is that the distribution is bimodal with 0% and 100% recoveries because unsecured loans are not easy to be recovered, whereas secured loans can be almost recovered by collateral. The unsecured loans tend to focus on 0% recovery, but the distribution of recovery rates is bimodal even for unsecured loans, as shown in Section 2. Therefore, we need to select the model which estimates the recovery rates with a bimodal distribution, and guarantees that the predicted values lies in the interval from 0 to 1.

Next, we introduce the methods for modelling in the previous studies. Dermine and Neto de Carvalho (2006) model recovery rates by fractional response regression model with log-log function which can map the interval. Bastos (2010) develops a nonparametric and nonlinear regression tree model. Itoh and Yamashita (2008) develop two types of models. At first, the binomial logistic regression is modeled for estimating 0% recovery or otherwise. Second, the ordered logistic regression is modeled using four categories; 0% recovery, more than 0% to less than 50%, 50% to less than 100%, and 100% and more. They mention the reasons they use the ordered logistic regression is that they want to estimate the model with categories of 0% and more than 100% recoveries because there are a lot of 0% recoveries and the upper bound of recovery rates is a time-dependent function. Kawada and Yamashita (2013) develop the two-stage LGD model, which consists of a binomial logistic regression model for the Pr(LGD>0) model and the Pr(Recovery) model. Tanoue, et al. (2017) develop the multi-stage LGD model, which consists of a logistic regression model for the Pr(Recovery) model, the Pr(LGD>0) model, and the logit-transformed OLS model for the LGD regression model. Ogi et al.(2015) model the ordered logistic regression using three categories; 0%, more than 0% to less than 100%, and 100% recoveries.

There are two types of methods for modelling the recoveries; fractional response regression with the dependent variable expressed by the recovery rates, and binomial or ordered logistic regression with the dependent variable expressed by mapping recovery rates into some categories. In our paper, we adopt the ordered logistic regression model with three categories as well as Ogi et al.(2015). The reasons are as follows.

(1) We find the high percentages of no recovery (0%) and full recovery (100%). It is important to estimate recovery rates precisely for the categories of 0% and 100% for a practical perspective because we can focus on recovering the 100% recovery loans and do nothing for 0% recovery loans at time of default. It is also easy to handle the ordered logistic regression in practice, and the method is familiar with credit risk management.

(2) Ogi et al.(2015) model the logit-transformed linear regression for the defaulted loans where recovery rates lie in the range between more than 0% and less than 100%, but the adjusted R-squared is 0.1. It fails to recognize the difference among the range. The reason is that the recovery rates are dependent on the work-out-process by banks, but it is complicated and difficult to model.

Therefore, we then construct the ordered logistic model as a categorical value for the range instead of modelling the recoveries as a continuous value.[2]

Based on the fact that the RR is bimodal at 100% and 0%, the ordered category consists of three categories; category 0 for RR=100%, category 1 for 0 %< RR<100% and category 2 for RR=0%, as

$$y_i = \begin{cases} 0 & (RR_i = 100\%) \\ 1 & (0\% < RR_i < 100\%) \\ 2 & (RR_i = \quad 0\%) \end{cases} .$$

The probabilities of each category are as follows:

$$\begin{aligned} P(y_i = 0) &= F(\alpha_1 + \boldsymbol{\beta}^{\boldsymbol{T}}\boldsymbol{X}_i) \\ P(y_i = 1) &= F(\alpha_2 + \boldsymbol{\beta}^{\boldsymbol{T}}\boldsymbol{X}_i) - F(\alpha_1 + \boldsymbol{\beta}^{\boldsymbol{T}}\boldsymbol{X}_i) \\ P(y_i = 2) &= 1 - F(\alpha_2 + \boldsymbol{\beta}^{\boldsymbol{T}}\boldsymbol{X}_i) \end{aligned}$$

[2] We do not examine the fractional regression, but the comparison between them is our future research.

$$F(x) = \frac{e^x}{1 + e^x}$$

where $\boldsymbol{X}_i$ represents the explanatory variable vector, α_1 and α_2 are constant terms, $\boldsymbol{\beta}$ is a regression coefficient vector, and F is a link function, which follows a logistic distribution.

The regression coefficients α_1, α_2 and $\boldsymbol{\beta}$ are estimated so as to maximize a likelihood function which is

$$L(\alpha_1, \alpha_2, \boldsymbol{\beta}) = \prod_{i \in B_0} F(\alpha_1 + \boldsymbol{\beta}^{\boldsymbol{T}} \boldsymbol{X}_i) \prod_{i \in B_1} \left(F(\alpha_2 + \boldsymbol{\beta}^{\boldsymbol{T}} \boldsymbol{X}_i) - F(\alpha_1 + \boldsymbol{\beta}^{\boldsymbol{T}} \boldsymbol{X}_i) \right) \prod_{i \in B_2} \left(1 - F(\alpha_2 + \boldsymbol{\beta}^{\boldsymbol{T}} \boldsymbol{X}_i) \right),$$

$$B_k = \{ i | y_i = k \}.$$

$\boldsymbol{\beta}^{\boldsymbol{T}} \boldsymbol{X}_i$ shows a score to evaluate each firm i in this paper.

$$SCORE_i = \boldsymbol{\beta}^{\boldsymbol{T}} \boldsymbol{X}_i$$

We employ Somers' D, which is a measure for prediction of classification, such that the large (small) $\beta^T X_i$ ($\mathrm{SCORE_i}$) leads to the high (low) recovery rates. The values lie in the range between 0% and 100%, and the large value expresses a high predictive power.

3.2 RR estimation models

In our paper, we develop the different RR estimation models at both appraisal time and default time respectively because we use the RR estimation for different purpose in practice.

(1) RR estimation model at appraisal time: Loan model

We construct the RR estimation model using financial accounting variables and attribute variables available when the loan is newly provided.[3] We call it "loan model" in this paper. Banks use the PD model to determine the loan decision or whether the loan is provided or not. However it is important to estimate the EL which is calculated based on the LGD, and banks have the needs of estimating the RR. It can be also used in order to determine the loan condition for example, appropriate loan interest rates, initial loan amount, together with making the decision of providing new loan. This is the first purpose of estimating the RR at appraisal time, and we can use it as examination support tool at providing new loan.

(2) RR estimation model at default time: After-default model

We construct the RR estimation model using attribute variables available at the time of default. We call it "after-default model" in this paper. We do not use financial accounting variables because banks can get obligors' financial data only at providing loan, and the values might be different at default. It is difficult for banks to update the financial data of obligors after providing small-business loan because it is overloaded for small-sized firms to offer the updated financial data.

Banks need to develop risk management strategy for defaulted loan because recovery amounts are small for small-business loan and therefore they implement the recovery operations efficiently. It can be done by prioritizing defaulted loans in order of the scores, calculated by the RR estimation model. This is the second purpose of estimating the RR at the time of default, and we can use it as risk management tool for improving the RR after default.

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[3] The discounting rate of the expected loss, the term structure of default, and that of recovery are also important elements to evaluate new loans. However, it lies outside the scope of our paper, and it is our future research.

We use different factors for estimating the RR for each model, because available data is partly different at different time points; time of providing new loans or default. In addition, we model the recoveries for sole proprietorships and industrial corporations, respectively. Factors may be different from each other for each type of firms. However, we develop the final models in reference to each other from a practical perspective.

3.3 Explanatory variables for estimating the RR

We summarize the explanatory variables for estimating recoveries employed in the previous studies in Table 3.

Table 3: Explanatory variable list in previous studies

<table>
<tr><th>Authors</th><th>CRT (*1)</th><th># of loans</th><th>RM (*2)</th><th>MDL (*3)</th><th colspan="2">Significant variables for p<0.1 (estimated sign)</th></tr>
<tr><td rowspan="2">Dermine and Neto de Carvalho (2006)</td><td rowspan="2">PRT</td><td rowspan="2">374</td><td>24</td><td rowspan="2">FR</td><td rowspan="2">Loan size (-), Collateral (+), Age of firm (+)</td><td>Year 1997 dummy (+)</td></tr>
<tr><td>48</td><td>Manufacturing sector (-), Trade sector (-)</td></tr>
<tr><td rowspan="2">Bastos (2010)</td><td rowspan="2">PRT</td><td rowspan="2">374</td><td>24</td><td rowspan="2">FR</td><td rowspan="2">Loan size (-), Collateral (+), Age of firm (+)</td><td>Personal guarantees (-), Rating (-), Years of relationship (+)</td></tr>
<tr><td>48</td><td>Manufacturing sector (-), Trade sector (-)</td></tr>
<tr><td rowspan="2">Itoh and Yamashita (2008)</td><td rowspan="2">JPN</td><td rowspan="2">645</td><td rowspan="2">24</td><td>BL</td><td rowspan="2">Year-on-year liabilities (-), Year-on-year ordinary profit (-), Number of employer (-), Second regional bank dummy (-), Financial stabilization special guarantee dummy (-), Collateral (land) dummy (-)</td><td>Cash-to-current-liabilities ratio (+), Construction industry dummy (-)</td></tr>
<tr><td>OL</td><td>Receivables turnover period (+)</td></tr>
<tr><td>Kawada and Yamashita (2013)</td><td>JPN</td><td>6,718</td><td>-</td><td>BL</td><td colspan="2">CRITS score (+), Collateral by real estate/EAD (+), Collateral by Commercial paper/EAD (+), Collateral by Deposit/EAD (+), Collateral by securities/EAD (+), Coverage by guarantee/EAD (+), EAD(Exposure at default) (-)</td></tr>
<tr><td rowspan="3">Tanoue et al. (2017)</td><td rowspan="3">JPN</td><td rowspan="3">8,732</td><td rowspan="3">-</td><td>MS1 (*4)</td><td rowspan="3">Collateral quota (real estate (+), commercial bills (+)), Credit guarantee quota (+)</td><td>Creditworthiness score (-), EAD (-)</td></tr>
<tr><td>MS2 (*4)</td><td>Collateral quota (deposits (+), marketable securities (+)), EAD (-)</td></tr>
<tr><td>MS3 (*4)</td><td>Collateral quota (deposits (+), marketable securities (+)), EAD (+)</td></tr>
<tr><td rowspan="2">Ogi et al. (2015)</td><td rowspan="2">JPN</td><td>5321 (*5)</td><td>36</td><td rowspan="2">OL</td><td colspan="2">Only variable categories shown (eight variables: two liquidity accounts, two safety accounts, firm age, loan amount, loan condition, commodity price)</td></tr>
<tr><td>1329 (*6)</td><td>36</td><td colspan="2">Only variable categories shown (five variables: Liquidity account, two safety accounts, quantitative index, loan condition)</td></tr>
</table>

*1 CRY (Country, PRT: Portugal, JPN: Japan), *2 RM(Recovery months), *3 MDL (Model, FR: Fractional Response regression, BL: Binomial Logistic regression, OL: Ordered Logistic regression, MS: Multi-Stage model using BL and ordinary least regression), *4 MS(Multi-stage LGD model, MS1: Pr(Recovery), MS2: Pr(LGD>0), MS3: LGD regression), *5 unsecured loans for sole proprietorships, *6 unsecured loans for industrial corporations

We find that the loan size, collateral, age of firm, personal guarantees are significant variables in Portugal. We confirm that collateral, guarantee, and firm age are also significant in Japan. We refer to the previous studies to select the variables. However our research cannot include the collateral which is one of the important explanatory variables because we need to estimate the recovery rates for unsecured loans. This is one of the different features between previous studies and our research.

We line up available attribute variables and financial accounting variables as candidate explanatory variables. Financial accounting variables are used only for the loan model as described before.

Attribute variables are divided into three types; (1) variables based on the loan application submitted from the obligor at providing loan (firm age, guarantee by business owner's family dummy, white tax return dummy, etc.), (2) variables determined by banks at providing loan (initial loan amount, loan of working capital dummy, etc.), (3) variables estimated by banks at default (exposure at default (EAD), EAD rate). The first and second attribute variables are available at providing loan, but they can be also used at default. The third attribute variables can be used only for the after-default model. Financial accounting variables are available on the balance sheets and profit and loss statements at providing loan. The variables calculated from financial accounts are also employed, such as real estate value minus debt amount, non-current asset minus non-current liabilities, and so on.

3.4 Procedure for variable selection

We use both financial accounting and attribute variables for the loan model, and attribute variables for the after-default model. We construct the final model with several variables because it is important to keep a handful of variables in order to increase the robustness of the model from a practical perspective. We implement the variable selection for the ordered logistic regression in the following steps.

Step 1: Conduct the ordered logistic simple regression analysis for each financial accounting variable because of a lot of variables. Calculate Somers' D and the significance probability (p-value). Select the variables with large Somers' D and lower p-value as candidates for Step 2.

Step 2: Conduct the cluster analysis with correlations between candidate variables. Select the representative variables with relatively large Somers' D if highly correlated variables are found. In selecting variables, we also refer to the opinion of practitioners in charge of financing loans.

Step 3: Narrow down both financial accounting and attribute variables for the loan model, and attribute variables for the after-default model using the stepwise method. Each model is constructed for sole proprietorships and industrial corporations, respectively.

Step 4: Examine the selected variables for each model constructed in Step 3. Exclude the variables which do not meet the sign condition, or have lower standardized estimates. Include the variables which are not selected in Step 3, but are essential from a practical perspective and statistically significant. Estimate the coefficients through the ordered logistic regression. Repeat this process until the best combination is selected, and construct the final model with several variables.

4. Estimating recovery rates

4.1 Variable selection process from Step 1 to Step 3

We line up available 290 variables which consist of 249 financial accounting variables, and 41 attribute variables as candidate explanatory variables. The amount of data used in the loan model is smaller than that in the after-default model because some amounts of financial accounting data used for the loan model were not available as electronic data. The number of data for each type of variables is as follows. We then construct each model for each type of firms using different data set. The number of attribute variables is the same as that of loans.

	Sole proprietorships	Industrial corporations
Attribute variables	29,772	37,156
Financial accounting variables	14,778	29,860

According to the procedure in Section 3.4, we show the variable selection process for each step.

In Step 1, we select some financial variables with high values of Somers' D as candidates for Step 2. In Step 2, we select 28 financial variables and 18 attribute variables in consideration with correlations between variables, as candidates for Step 3. Abbreviated names described in Tables A.1 and A.2 are employed for explanatory variables hereafter. Refer to Appendix A for details.

In Step 3, we select the both financial accounting variables and attribute variables for the loan model, and attribute variables for the after-default model by the stepwise method using variables listed by Step 2. The number of selected variables for each model is as follows.

	Loan model				After-default model	
Model	Sole proprietorships(S.P.)		Industrial corporations(I.C.)		S.P.	I.C.
Number of variables	Attr. V.	Fin. V.	Attr. V.	Fin. V.	Attr. V.	Attr. V.
	9	7	9	10	12	11

Table 4 shows the results of ordered logistic regressions by the stepwise method.

The neglog transformation[4] is applied to initial loan amount (A03), real-estate value minus debt amount (F27), and non-current assets minus non-current liabilities (F28). The EAD rate is calculated as the initial loan amount divided by expose at default (EAD).

We find four attribute variables are commonly selected for each model. The guarantee by business owner's family dummy has the largest estimates on average, and gives larger effects to recoveries at the time of providing new loan and default. It is a reasonable result because the recoveries are related with the guarantee. The second effective variable is firm age. The higher the firm age is, the higher the RR after default is. The estimates of the models for industrial corporations are larger than those for sole proprietorships.

Medical industry dummy is selected, and has a positive sign because there may be a lot of obligors who have the ability of repayment in medical industry. Repayment period is selected, and has a negative sign because the longer the repayment period is, the higher EAD rate might be and the lower the RR after default is. However, the estimates of medical industry dummy and repayment period are relatively small.

[4] The neglog transformation can be used for transforming the both positive and negative values to logarithms as follows.

$$neglog(x) = \begin{cases} -\log(1-x), x \le 0 \\ +\log(1+x), x > 0 \end{cases}$$

Table 4: Results of ordered logistic regressions by stepwise method in Step 3

		Loan model				After-default model				expected
	Variables	S.P.(*1)		I.C.(*2)		S.P.(*1)		I.C.(*2)		sign
A01	Firm age (*3)	0.07	***	0.13	***	0.05	***	0.12	***	+
A02	Number of employers					−0.05	***			−
A03	Initial loan amount	−0.11	***					0.33	***	−
A04	Guarantee dummy	0.20	***	0.16	***	0.20	***	0.18	***	+
A05	Manufacturing I.dummy	−0.02	**			−0.01	**	−0.02	***	+/−
A06	Construction I.dummy					−0.03	***	−0.03	***	+/−
A09	Medical I.dummy	0.03	***	0.02	***	0.03	***	0.01	***	+/−
A10	Service I.dummy			0.01	**					+/−
A11	Real estate I.dummy	0.03	***							+/−
A12	Transport I.dummy							−0.02	***	+/−
A13	Working capital dummy			−0.03	***	−0.03	***	−0.04	***	−
A14	Repayment period	−0.05	***	−0.03	***	−0.03	***	−0.02	***	+/−
A15	White tax return dummy	−0.05	***	−0.02	**	−0.02	***			−
A16	Owner's age (*3)	−0.04	***	0.07	***	−0.02	**	0.06	***	+/−
A17	EAD rate					−0.37	***			−
A18	EAD					−0.08	***	−0.52	***	−
F01	Sales	−0.04	***							+
F04	Gross profit			−0.02	***					+
F06	Labor cost			0.04	***					−
F07	Depreciation cost	0.04	***							+
F08	Operating profit	−0.03	***	−0.03	***					+
F09	Non-operating expenses	−0.03	**							−
F10	Interest expenses			−0.09	***					−
F13	Cash and deposits	0.03	***							+
F16	Other current assets			−0.03	***					+
F19	Current liabilities			0.03	***					−
F20	Accounts payable			0.03	***					−
F23	Long-term debt	0.04	***							−
F25	Monthly repayment			0.04	***					−
F26	Labor costs for M			−0.03	***					−
F27	Real-estate minus debt	0.21	***							+
F28	Non-current A minus L			0.02	***					+
	Number of loans	14,778		29,860		29,772		37,156		
	Somers' D	29%		22%		39%		33%		

$p < 0.01$: ***, $p < 0.05$: **, $p < 0.1$: *

*1 S.P.: Sole proprietorships, *2 I.C.: Industrial corporations, *3 Firm age and owner's age at providing loan are used in the loan model, whereas those at default are used in the after-default model.

4.2 Constructing the final model in Step 4

We construct the final models, based on the results of Step 3 (Table 4). We examine the variables in each model, and select explanatory variables in consideration of whether they are also used in other model and other type of firms, because each model is related to each other. We confirm that all selected variables are significant for each final model, as shown in Table 5

Table 5: Final models

		Loan model		After-default model		expected
	Variables	S.P.(*1)	I.C.(*2)	S.P.(*1)	I.C.(*2)	sign
A01	Firm age	0.05 ***	0.14 ***	0.02 **	0.13 ***	+
A03	Initial loan amount	−0.12 ***	−0.04 ***			−
A04	Guarantee dummy	0.19 ***	0.14 ***	0.18 ***	0.19 ***	+
A13	Working capital dummy		−0.02 ***	−0.03 ***	−0.04 ***	−
A15	White tax return dummy	−0.05 ***	−0.01 **			−
A17	EAD rate			−0.38 ***	−0.33 ***	−
F27	Real-estate minus debt	0.21 ***				+
F28	Non-current A minus L		0.04 ***			+
	Number of loans	14,778	29,860	29,772	37,156	
	Somers' D	28%	19%	39%	32%	

*1 S.P.: Sole proprietorships, *2 I.C.: Industrial corporations

We explain the selection process below.

(1) Loan model for sole proprietorships

The stepwise method selects nine attribute variables and seven financial variables in Step 3. All variables are significant, but we find four variables which have large estimates; firm age, initial loan amount, guarantee by business owner's family dummy, and real-estate value minus debt amount. At first, these four variables are included in final models because they meet reasonable sign conditions and bankers can be convinced to select them, based on practical experience. Details are shown from (1) to (4) in Section 4.3.

Next additional variables are examined for the significant variables with large estimates. We select white tax return dummy as the explanatory variable from a practical perspective. The estimate is 0.05, and it is also significant in the loan model for industrial corporations. A tax return reports income, expenses, and other financial accounts, and Japanese tax returns are classified into the white and blue tax returns. It is unnecessary for the white tax return to keep books more strictly than the blue tax return. We assume the business owner who submits a form of a white tax return tends to be unfamiliar with financial aspects, and this causes a negative effect.

The Somers' D of the estimated model by Step 3 is 29%, whereas the Sommers' D of the final model is 28%. We can model the recoveries with less numbers of explanatory variables than the estimated model by Step 3.

(2) Loan model for industrial corporations

The stepwise method selects eight attribute variables and ten financial variables in Step 3. All variables are significant, but we find two variables with large estimates; firm age and guarantee by business owner's family dummy. At first, these two variables are included in the final models. Second, we select the white tax return dummy as well as the loan model for sole proprietorships.

We also select non-current assets minus non-current liabilities for industrial corporations. It is similar to real-estate value minus debt amount which is selected with the largest estimate for sole proprietorships. This causes a positive effect due to the ability for recoveries. In addition, we select the working capital dummy, which causes a negative effect because it is used on a regular basis and it is difficult to use in order to repay the loan. We find it is also significant in the after-default model. Therefore we select these two variables. Finally, we examine the initial loan amount even though it is not selected by the stepwise method. The reason is that it is expected to be an important factor for recoveries, and it is selected in the loan model for sole proprietorships. Details are shown in Section 4.3(4). We confirm that the sign is negative and it is statistically significant with the p-value of less than 1% in the final model. Therefore, we select it in the final model.

The Somers' D of the final model is 19%, which is smaller than 22% of the estimated model by Step 3. We have several unreasonable sign conditions for financial accounting variables, for example, negative signs for gross profit and operating profit, and positive signs for current liabilities and average monthly principal repayment for long-term debt. We select the final model with six variables though we can select more explanatory variables in order to derive the higher Somers' D,

(3) After-default model for sole proprietorships

The stepwise method selects twelve attribute variables in Step 3. All variables are significant, but we find two variables which have large estimates; guarantee by business owner's family dummy and EAD rate. At first, these two variables are included in the final models. Second, we select the firm age as well as other models. Finally, we also select working capital dummy due to the same reason as shown abovementioned. The values of Somers' D of both the estimated model by Step 3 and the final model are 39%. We can model the recoveries with four explanatory variables.

(4) After-default model for industrial corporations

The stepwise method selects eleven attribute variables in Step 3. All variables are significant, but we find four variables which have large estimates; firm age, guarantee by business owner's family dummy, initial loan amount, and EAD. The negative sign is expected for initial loan amount, but we find the value is positive. We then remodel the recoveries by stepwise method except initial loan amount. EAD rate gets selected alternatively as a significant variable, and the standardized estimate is -0.10, whereas the estimate of EAD becomes -0.25.[5] The EAD rate is calculated using EAD, and therefore we need to select either to avoid multicolinearity. The estimate of EAD is larger than that of EAD rate, but we select the EAD rate because the model for sole proprietorships has the largest EAD rate, and we can also get the largest estimate in the final model. Additional explanations are shown in Section 4.3(5). We also select working capital dummy due to the same reason as shown abovementioned. Then, we include the four variables in the final model. The Somers' D of the estimated model by Step 3 is 33%, whereas the Sommers' D of the final model is 32%. We can model the recoveries with less numbers of explanatory variables than the estimated model by Step 3.

4.3 Additional explanation for selected explanatory variables

(1) Guarantee by business owner's family dummy

Guarantee by business owner's family dummy is statistically significant, and it has large negative standardized estimates in each model for each type of firms. Each definition of personal guarantee variable is different for previous studies, but it is also significant. We find it is a key factor for unsecured loans, instead of collateral for secured loan.

(2) Firm age

The higher the obligor's firm age is, the higher the RR after default is. Especially, it is more effective for industrial corporations than sole proprietorships. It can be explained as follows. The explanatory powers of financial variables showing long-term safety evaluated by non-current assets

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[5] Due to space limitation, we omit to show the result of the model excluding initial loan amount.

and liabilities in the balance sheet for industrial corporations are not so high as those for sole proprietorships.[6] Specifically, the standardized estimate of non-current assets minus non-current liabilities is 0.04, and it becomes lower than that of real-estate value minus debt amount for sole proprietorships which estimate is 0.21. The reason is that the business owner's assets and liabilities are not taken into consideration for industrial corporations. Instead, the estimate of the firm age is 0.14 for the loan model, it can be thought as the proxy of assets and liabilities of business owner.[7] On the other hand, firm age has a significant but low estimate for sole proprietorships, because the additional effect by firm age is small by the effect of real-estate value minus debt amount with a large estimate.

(3) Real-estate value minus debt amount in the loan model for sole proprietorships

The higher the obligor's real-estate value minus debt amount is, the higher the RR after default is. We examine the Somers' D in a simple regression for each financial variable. The values of Somers' D are large for long-term safety financial variables evaluated by non-current assets and liabilities in the balance sheet.[8] The values of financial variables cannot be available at time of default, but they might have varied only slightly after the time of providing new loans until the default time,[9] and therefore the variable affects the RR after default.

The recovery rates for unsecured loans to sole proprietorships depend on the real estate value of the obligors. This shows the real estate is expected to be utilized for repayment although it is not the collateral of the unsecured loan. If so, why do the obligors borrow the unsecured loan regardless of the fact that the interest rate is offered higher than the secured loan? The answers are empirically as follows.

1) The obligors feel uncomfortable offering the real estate as collateral, because it is more likely to be home of owner's own, and it causes the psychological stress.

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[6] There are many small-sized firms even for industrial corporations, and frequently the non-current assets and liabilities held by business owner are larger than those owned by corporations. The Japanese Financial Services Agency(2015a) publishes the inspection manual in 2015, which is utilized by the inspector for inspection by financial institutions. Different from a large company, the properties of business owner in a small firm are not clearly separated from those of the firm. Furthermore, in classifying the obligors of small firms in self-assessment, it is said that the business owner has repayment ability if he has non-current assets such as real estate.

[7] Ogi, Toshiro and Hibiki (2014) proposed the effectiveness of firm age in the PD estimation model for small firms, and showed the firm age might be a proxy variable of personal assets and liabilities of the business owner.

[8] The values of Somers' D are small for financial variables indicating profitability in the profit and loss statement and short-term solvency evaluated by current assets and liabilities in the balance sheet. The profitability and short-term solvency may have already declined at the time of default even if they are high at the time of providing new loans. The default is defined as the overdue destination of three months or more, and the RR is not affected after default.

[9] If the real-estate value minus debt amount is almost kept after the time of providing new loans until the default time, we might have a chance of adding the variable to the current after-default model with four attribute variables for sole proprietorships. However, the number of data for financial variables is 14,778, whereas the number of data for attribute variables is 29,772. Due to space limitation, we omit the details but show the results of the after-default model using 14,778 loans. The value of Somers' D for the four-variable model is 31%, whereas the value for five-variable model is 35%. This shows this variable is also effective for the after-default model even though it is not a value available at default time. But the value of Somers' D is 39% for the four-variable model using 29,772 loans. Therefore, we adopt the four-variable model using only attribute variables in our paper. It is our future research to validate the five-variable model after we store more default and recovery data.

2) The obligors hope to avoid more effort and time in order to borrow the small-sized loan because it takes a long time from loan request until money receipt when the collateral is offered.

(4) Initial loan amount in the loan model

The smaller the initial loan amount is, the higher the RR after default is, because the EAD tends to be small, although it depends on the actual repayment period. The repayment is possible even after default, especially for JFC-Micro because there are a lot of obligors with the small amount of loan, and the repayment plan can be moderate. As shown in Table 1, the average EAD for loans to sole proprietorships is as little as 1.92 million yen. According to the family income and expenditure survey published by the Ministry of Internal Affairs and Communications(2017) of the Government of Japan, the monthly consumption expenditure in 2017 is 0.24 million yen on average, or 2.92 million yen on an annual basis. The average EAD of 1.92 million yen for loans to sole proprietorships is approximately two-thirds of average household consumption expenditure.

(5) EAD rate in the after-default model

The lower the EAD rate is, the higher the RR after default is. The EAD rate is obtained by dividing the EAD by the initial loan amount, and it shows how much is repaid to the initial loan amount by the default time. As a preliminary analysis before constructing the model, Somers' D is examined by conducting a simple regression analysis with the EAD and initial loan amount, respectively. The explanatory powers of both variables are high. The three variables including EAD rate are also highly correlated each other. The highest Somers' D is obtained by the simple regression with the EAD rate. We find the EAD rate is a more effective variable than other two variables, and therefore we adopt the EAD rate in the model.

4.4 Comparing our results with preceding works except Japanese banks

The results are obtained for Japan Finance Corporation (JFC), and therefore we mention whether these are specific or common to banks in the world except Japan by comparing with preceding works to small sized firms.

The distribution of recovery rates is bimodal with 0% and 100% recoveries in previous studies. The form is basically the same as the distribution in JFC. The previous studies include both unsecured and secured loans, but our research includes only unsecured loans. Unsecured loans are not easy to be recovered, whereas secured loans can be almost recovered by collateral. Therefore, we find the percentage of full recovery (100% RR) is larger than no recovery (0% RR) in the previous studies, whereas the full recovery is smaller than no recovery as shown in Figure 3 in our research.

Next, we compare the significant variables used in the models. We have only two previous papers in Portugal [Dermine and Neto de Carvalho(2006), Bastos(2010)] available for comparison, as shown in Table 3. The significant variables are the loan size, collateral, age of firm, personal guarantees. The preceding models are constructed at default, and therefore we compare them with variables in the after-default model. Firm age and guarantee by business owner's family (personal guarantee) are common to both the JFC and banks of Portugal. Our specific variable is exposure rate at default, instead of the loan size for banks of Portugal. Our models do not include the collateral because of unsecured loans.

5. Evaluation of the RR model

5.1 Model for loans to sole proprietorships

We divide all defaulted loans into four classes equally in descending order of the score ($\boldsymbol{\beta}X_i$), and give the ratings of ‘A’ to ‘D’. We test the usefulness in practice by examining the order of the actual RR by rating 'A' > 'B' > 'C' > 'D'. The larger the difference between the RR of ratings 'A' and 'D' is, the more useful the model is in practice. When we use the model as a risk management tool after default, the greater the difference of the actual RR between the ratings 'A' and 'D', the more effective the effects of the management policy and the recovery operations to the obligors in order of the score are.

We show the cumulative distributions of actual RRs for each rating in Figure 4. These graphs are generated by the eleven kinds of percentiles (0%, 1%, 5%, 10%, 25%, 50%, 75%, 90%, 95%, 99%, 100%), which are linearly interpolated for simplicity. It turns out that the order of the actual RR by rating in a proper order. The actual RRs by the after-default model is larger than those by the loan model because we find more effective factors after default than the time of providing loans.

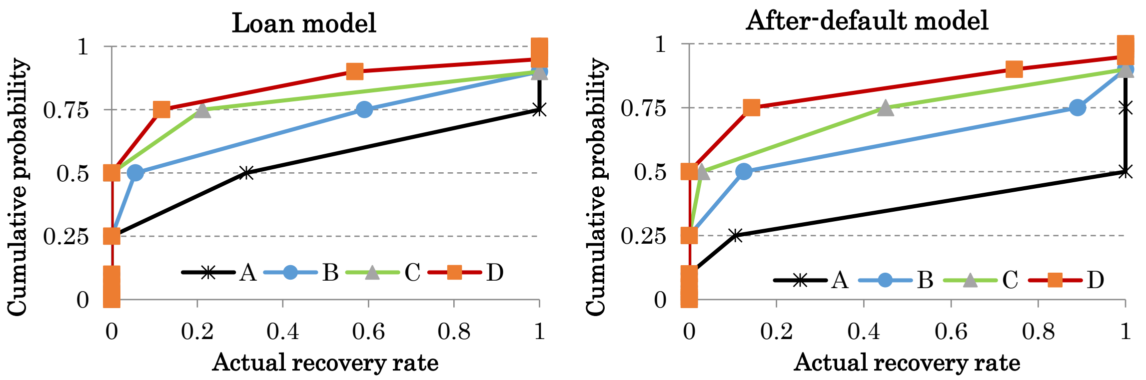

Figure 4: Cumulative distribution of actual RR for loans to sole proprietorships

Table 6 shows the mean actual RR during five years for each rating.[10] The order of RR by rating is '47% for A-rating' > '30% for B' > '20% for C' > '14% for D' in the loan model. The order of RR by rating is '64% for A-rating' > '36% for B' > '26% for C' > '16% for D' in the after-default model. The actual RRs of each rating are given in a proper order in both models. When calculating the difference between the mean actual RR of ratings 'A' and 'D', it is as large as 33% points (=47%−14%) in the loan model and 48% point (=64%−16%) in the after-default model. We confirm that the performance level is sufficiently acceptable in practice.

Next, we perform the Shirley-Williams' multiple comparison test for comparing the difference among multiple ratings, because the recovery rates are expected to be ranked by rating, and the Shirley-Williams' test is suitable for the credit rating. In our paper, the Shirley-Williams' test is examined sequentially for three dose groups which correspond to B, C, D-ratings, and the A-rating corresponds to the control group. The comparison of A-rating with (B, C, D)-ratings is called "Level 3", that with (B, C)-ratings is called "Level 2", and that with B-rating is called "Level 1". The results show that the p-values are less than 1% for three levels, and the ranks of ratings are statistically significant at 1% level.

In addition, we also perform the Kruskal-Wallis test and the Mann-Whitney U test to examine the difference among rating groups. The Kruskal-Wallis test can be employed for comparing the

[10] We evaluate the models using the mean actual RRs for each rating in our paper because the RRs of each defaulted loan are equally weighted for constructing the estimation model. However, evaluating the EAD-weighted actual RRs is also important in practice because the actual amounts of recoveries are dependent on the EAD. For reference, we show the results of the EAD-weighted actual RRs in Appendix B. The EAD-weighted actual RRs are consistently undervalued, compared with the mean actual RRs.

difference between ratings, and the Mann-Whitney U test can be employed to examine the difference between two groups on the combinations of ratings 'A' and 'B', ratings 'B' and 'C', and ratings 'C' and 'D'. The Kruskal-Wallis test shows that the p-values are less than 1%, and there are statistically significant at 1% level. Mann-Whitney U test also shows that the p-values are less than 1% in all combinations, and we find there are significantly different among all combinations.

Table 6: Mean actual RR for loans to sole proprietorships

Loan model			After-default model		
Rating	N	Actual RR	Rating	N	Actual RR
A	3,695	47%	A	7,441	64%
B	3,694	30%	B	7,445	36%
C	3,694	20%	C	7,450	26%
D	3,695	14%	D	7,436	16%
All	14,778	28%	All	29,772	36%
Shirley-Williams' test: p-value < 0.01			Shirley-Williams' test: p-value < 0.01		
Kruskal-Wallis test: p-value < 0.01			Kruskal-Wallis test: p-value < 0.01		
Mann-Whitney U test for A&B, B&C, and C&D: p-value < 0.01			Mann-Whitney U test for A&B, B&C, and C&D: p-value < 0.01		

Next, we examine the robustness in time series by checking the order of the actual RR by fiscal years of default and providing loan. First, Table 7 shows the results by fiscal year of default. Every fiscal year of default, the actual mean RR of each rating is in a proper order. We omit the results due to space limitation, but the values of eleven percentiles are also in a proper order. Therefore it is robust by year of default in time series.

Table 7: Mean actual RR by fiscal year of default for sole proprietorships

Loan model						After-default model					
Fiscal year (N)		2008 (1,662)	2009 (3,596)	2010 (4,430)	2011 (5,090)	Fiscal year (N)		2008 (6,740)	2009 (8,028)	2010 (7,393)	2011 (7,611)
A		34%	36%	46%	60%	A		63%	62%	64%	68%
B		18%	23%	29%	40%	B		32%	32%	36%	45%
C		12%	13%	18%	29%	C		21%	23%	26%	34%
D		8%	11%	14%	17%	D		12%	15%	15%	20%
All		20%	21%	26%	36%	All		35%	33%	34%	41%
S-W: p-value		< 0.01	< 0.01	< 0.01	< 0.01	S-W : p-value		< 0.01	< 0.01	< 0.01	< 0.01
K-W: p-value		< 0.01	< 0.01	< 0.01	< 0.01	K-W : p-value		< 0.01	< 0.01	< 0.01	< 0.01
M-W U: p-value	A&B	< 0.01	< 0.01	< 0.01	< 0.01	M-W U : p-value	A&B	< 0.01	< 0.01	< 0.01	< 0.01
	B&C	< 0.01	< 0.01	< 0.01	< 0.01		B&C	< 0.01	< 0.01	< 0.01	< 0.01
	C&D	0.41	0.15	0.10	< 0.01		C&D	< 0.01	< 0.01	< 0.01	< 0.01

The Shirley-Williams' multiple comparison test shows that the p-values are less than 1% every fiscal year of default. We find that it achieves a sufficiently acceptable level in practice. The Kruskal-Wallis test also shows that the p-values are less than 1%. On the other hand, the Mann-Whitney U test shows the p-value is less than 1% except for three combinations between the ratings C and D of FY2008 to FY2010 in the loan model. The combinations of p-values are more than 1%, or 0.41, 0.15, and 0.10, respectively. These mean actual RRs and the values of percentiles are in a proper order, but it is not possible to reject the null hypothesis that there are no differences between ratings. This may be because of the small number of observations by classifying samples by fiscal year of default or rating.

Table 8 shows the results by fiscal years of providing loan. They range from the 1990's to the FY2011, but due to the space limitation, we show the results of only FY2007 to FY2009 with many observations. Every fiscal year of providing loan, the mean actual RR and the value of eleven percentiles of each rating is in a proper order. We confirm the robustness by year of providing loan in time series.

Table 8: Mean actual RR by fiscal year of providing loan for sole proprietorships

Loan model					After-default model				
Fiscal year		2007	2008	2009	Fiscal year		2007	2008	2009
(N)		(4,261)	(5,111)	(3,731)	(N)		(5,610)	(6,298)	(4,384)
A		44%	47%	51%	A		64%	64%	87%
B		28%	30%	31%	B		35%	46%	53%
C		19%	20%	21%	C		21%	27%	36%
D		15%	14%	14%	D		11%	14%	17%
All		32%	25%	27%	All		31%	26%	28%
S-W : p-value		< 0.01	< 0.01	< 0.01	S-W : p-value		< 0.01	< 0.01	< 0.01
K-W : p-value		< 0.01	< 0.01	< 0.01	K-W : p-value		< 0.01	< 0.01	< 0.01
M-W U: p-value	A&B	< 0.01	< 0.01	< 0.01	M-W U: p-value	A&B	< 0.01	< 0.01	< 0.01
	B&C	< 0.01	< 0.01	< 0.01		B&C	< 0.01	< 0.01	< 0.01
	C&D	0.08	< 0.01	< 0.01		C&D	< 0.01	< 0.01	< 0.01

The Shirley-Williams' multiple comparison test and the Kruskal-Wallis test show that the p-values are less than 1% every fiscal year of providing loan. The Mann-Whitney U test shows the p-values are less than 1% except for one combination of the ratings 'C' and 'D' of FY2007 in the loan model, which p-value is 0.08.

5.2 Model for loans to industrial corporations

We show the cumulative distributions of actual RRs for each rating in Figure 5. It also turns out that the order of the actual RR by rating in a proper order as well as Figure 4.

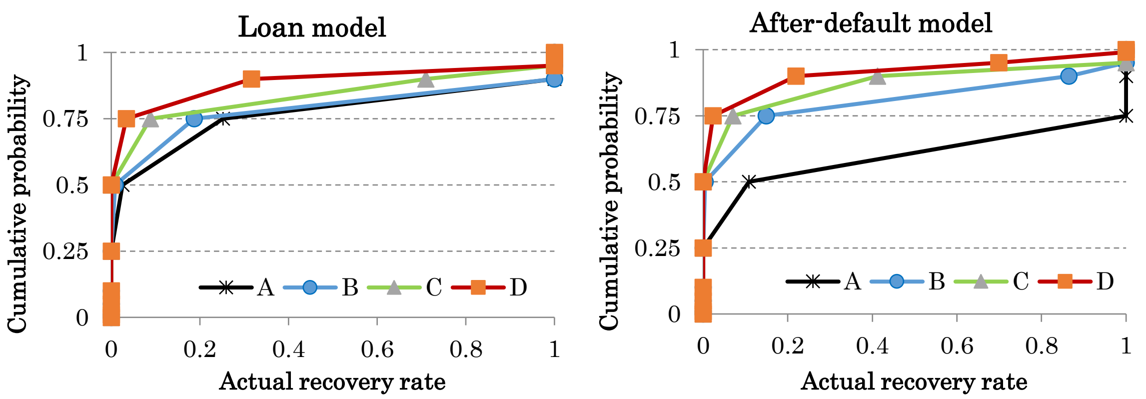

Figure 5: Cumulative distribution of actual RR for loans to industrial corporations

Table 9 shows the mean actual RR for each rating in the models for loans to industrial corporations. The mean actual RRs of each rating in both loan model and after-default model are in a proper order. When calculating the difference between the RR of ratings 'A' and 'D', it is as large as 12% points in the loan model and 31% points in the after-default model. We confirm that the level is sufficiently acceptable in practice. However, the difference between the rating ‘A’ and ‘D’ in the loan model is 12% points, which is smaller than that of the model for loans to sole proprietorships. The reason is that non-current assets and liabilities of business owners cannot be taken into consideration in the model for loans to industrial corporations.

The Shirley-Williams’ test shows that the p-values are less than 1%, and the ranks of ratings are statistically significant at 1% level. The Kruskal-Wallis test and Mann-Whitney U test also show that the p-values are less than 1%. We find that there are statistically significant differences at 1% level between rating groups.

Table 9: Mean actual RR for loans to industrial corporations

Loan model			**After-default model**		
Rating	N	Actual RR	Rating	N	Actual RR
A	7,465	22%	A	9,290	39%
B	7,466	20%	B	9,288	17%
C	7,464	14%	C	9,292	12%
D	7,465	10%	D	9,286	8%
All	29,860	17%	All	37,156	19%
Shirley-Williams' test: p-value < 0.01			Shirley-Williams' test: p-value < 0.01		
Kruskal-Wallis test: p-value < 0.01			Kruskal-Wallis test: p-value < 0.01		
Mann-Whitney U test for A&B, B&C, and C&D: p-value < 0.01			Mann-Whitney U test for A&B, B&C, and C&D: p-value < 0.01		

Tables 10 and 11 show the actual RR by fiscal years of default and providing loans, respectively. Every fiscal year of default and providing loans, the actual RR of each rating is in a proper order. We confirm the robustness in time series. Likewise, the Shirley-Williams’ test shows that the p-values are less than 1% in all years and the ranks of ratings are statistically significant at 1% level. The Kruskal-Wallis test and Mann-Whitney U test show that the p-values are less than 1% except for two combinations in the after-default model in Tables 11. We find that these are enough level to use in practice.

Table 10: Mean actual RR by fiscal year of default for loans to industrial corporations

Loan model						**After-default model**					
Fiscal year (N)		2008 (6,823)	2009 (7,772)	2010 (7,432)	2011 (7,833)	Fiscal year (N)		2008 (9,131)	2009 (9,950)	2010 (8,914)	2011 (9,161)
A		21%	21%	21%	25%	A		40%	42%	36%	36%
B		18%	19%	20%	22%	B		15%	17%	16%	19%
C		14%	14%	14%	16%	C		10%	10%	12%	15%
D		8%	9%	10%	13%	D		7%	8%	7%	11%
All		15%	16%	16%	19%	All		18%	19%	18%	21%
S-W : p-value		< 0.01	< 0.01	< 0.01	< 0.01	S-W : p-value		< 0.01	< 0.01	< 0.01	< 0.01
K-W : p-value		< 0.01	< 0.01	< 0.01	< 0.01	K-W : p-value		< 0.01	< 0.01	< 0.01	< 0.01
M-W U : p-value	A&B	< 0.01	< 0.01	< 0.01	< 0.01	M-W U : p-value	A&B	< 0.01	< 0.01	< 0.01	< 0.01
	B&C	< 0.01	< 0.01	< 0.01	< 0.01		B&C	< 0.01	< 0.01	< 0.01	0.02
	C&D	< 0.01	< 0.01	< 0.01	< 0.01		C&D	< 0.01	< 0.01	< 0.01	0.08

Table 11: Mean actual RR by fiscal year of providing loans for loans to industrial corporations

Loan model					After-default model				
Fiscal year (N)		2007 (8,221)	2008 (7,558)	2009 (4,955)	Fiscal year (N)		2007 (9,339)	2008 (8,456)	2009 (5,457)
A		20%	15%	20%	A		28%	25%	30%
B		19%	13%	16%	B		17%	14%	20%
C		13%	10%	12%	C		11%	10%	14%
D		10%	8%	8%	D		8%	7%	8%
All		15%	11%	14%	All		15%	12%	14%
S-W test : p-value		< 0.01	< 0.01	< 0.01	S-W test : p-value		< 0.01	< 0.01	< 0.01
K-W test : p-value		< 0.01	< 0.01	< 0.01	K-W test : p-value		< 0.01	< 0.01	< 0.01
M-W U: p-value	A&B	< 0.01	< 0.01	< 0.01	M-W U: p-value	A&B	< 0.01	< 0.01	< 0.01
	B&C	< 0.01	< 0.01	< 0.01		B&C	< 0.01	< 0.01	< 0.01
	C&D	< 0.01	< 0.01	< 0.01		C&D	< 0.01	< 0.01	< 0.01

6. Out-of-sample tests

We conduct out-of-sample tests for 14,624 loans (6,305 loans to sole proprietorships and 8,319 loans to industrial corporations) defaulted in FY2012, based on the models estimated in Section 4, using the 66,928 defaulted loans from FY2008 to FY2011. The quartile points of scores calculated by the estimated RR model in Section 4 are given to the thresholds for ratings. The ratings of out-of-samples are given based on the scores calculated by the model. Therefore the number of defaulted loans in each rating group are different each other, and there are some groups where the number of samples is much smaller than other groups. This is an important point different from the in-sample test in Section 5. We examine the consistency between the order of actual RR and ratings.

We also perform three kinds of statistical tests as well as the in-sample test. However, we need to pay attention to the fact that the results are likely to be greatly affected by the different numbers of samples in each rating group.

6.1 Model for loans to sole proprietorships

We show the cumulative distributions of actual RRs for each rating in Figure 6. The order of the actual RR by rating is in a proper order. The actual RRs by the after-default model are larger than those by the loan model, especially for 'A'-rating.

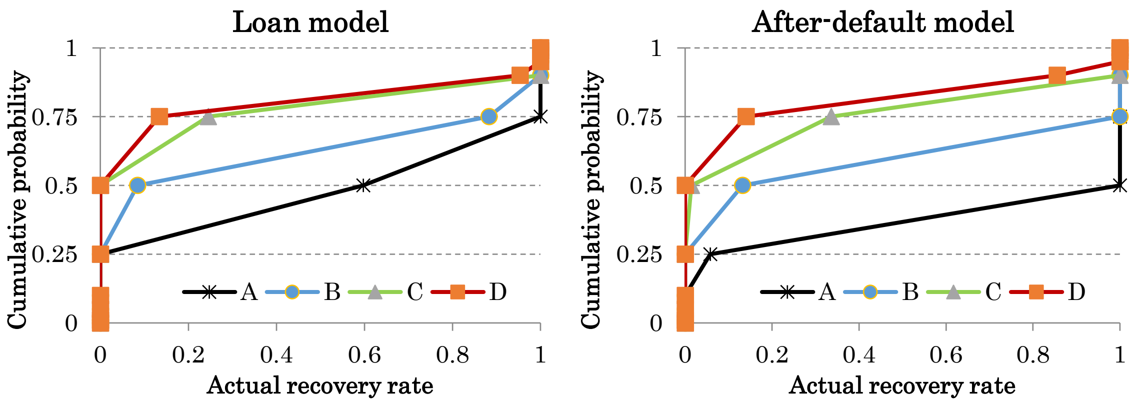

Figure 6: Cumulative distribution of actual RR for loans to sole proprietorships

We show the results of mean actual RR for loans to sole proprietorships defaulted in FY 2012 in Table 12. We find the actual RRs of each rating are also in proper orders for out-of-sample data as well as in-sample data used in estimating the model. The differences between the mean actual RR of ratings 'A' and 'D' are 37% points in the loan model and 48% point in the after-default model, and these are almost the same as the in-sample cases in Table 6.

The Shirley-Williams' test show that the p-values are less than 1% in both loan model and after-default model, and the ranks are statistically significant at 1% level between rating groups. The Kruskal-Wallis test and Mann-Whitney U test also show that the p-values are less than 1%.

Table 12 : Mean actual RR for loans to sole proprietorships defaulted in FY2012

	Loan model			After-default model		
	N	(ratio)	Actual RR	N	(ratio)	Actual RR
A	1,017	(22%)	53%	1,258	(20%)	64%
B	1,041	(23%)	34%	1,729	(27%)	39%
C	1,149	(25%)	22%	1,841	(29%)	24%
D	1,357	(30%)	16%	1,477	(23%)	16%
All	4,564	(100%)	30%	6,305	(100%)	34%
Shirley-Williams' test	p-value < 0.01			p-value < 0.01		
Kruskal-Wallis test	p-value < 0.01			p-value < 0.01		
Mann-Whitney U test for A&B, B&C, and C&D	p-value < 0.01			p-value < 0.01		

We show the results of actual RR for defaulted loans in FY2012, provided from FY2007 to FY2010 respectively in Table 13.

Table 13 : Mean actual RR for loans defaulted in FY2012, provided from FY2007 to FY2010

		Loan model				After-default model			
FY		2007	2008	2009	2010	2007	2008	2009	2010
A		62% (200)	53% (204)	51% (235)	57% (206)	67% (324)	59% (235)	63% (99)	56% (28)
B		42% (166)	38% (173)	36% (241)	30% (259)	25% (250)	39% (491)	42% (549)	48% (239)
C		31% (61)	28% (239)	22% (281)	21% (300)	17% (52)	18% (289)	20% (513)	28% (680)
D		27% (50)	24% (267)	17% (370)	14% (343)	1% (4)	11% (55)	11% (182)	13% (387)
All		47% (477)	35% (883)	29% (1,127)	27% (1,108)	46% (630)	36% (1,070)	31% (1,343)	28% (1.334)
S-W test : p-value		< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	0.22(a)
K-W test: p-value		< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01
M-W U test: p-value	A&B	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	0.44
	B&C	0.12	0.05	< 0.01	< 0.01	0.26	< 0.01	< 0.01	< 0.01
	C&D	0.67	0.15	0.04	0.09	0.16	0.17	0.01	< 0.01

* The number of defaulted loans is in the parenthesis. (a) Level 1:p=0.22, Level 2&3: p<0.01

We find the mean actual RRs of each rating are in a proper order. The Shirley-Williams' test shows that the ranks except FY2010 in after-default model are statistically significant at 1% level. The p-value of Level 1 of FY2010 in after-default model is 0.22, because the number of A-rated loans is much smaller than that of B-rated loans. The Kruskal-Wallis test shows that there are statistically significant differences at 1% level. There are thirteen combinations which are statistically significant at 1% level in the Mann-Whitney U test, but there are seven combinations which are not significant at 10% level. This is likely to be caused by data deficiency. For example, there are only fifty defaulted loans for D-rating in the loan model. We need more out-of-sample periods in order to perform the Mann-Whitney U test for each year of providing loans because the number of data varies widely among different rating groups in out-of-sample data.[11] However, the results of three kinds of statistical tests show that the models are significant as in Table 12. Therefore we have enough dataset to implement out-of-sample test if the dataset is not categorized into four years of providing loans.

6.2 Model for loans to industrial corporations

We show the cumulative distributions of actual RRs for each rating in Figure 7. The order of the actual RR by rating is in a proper order.

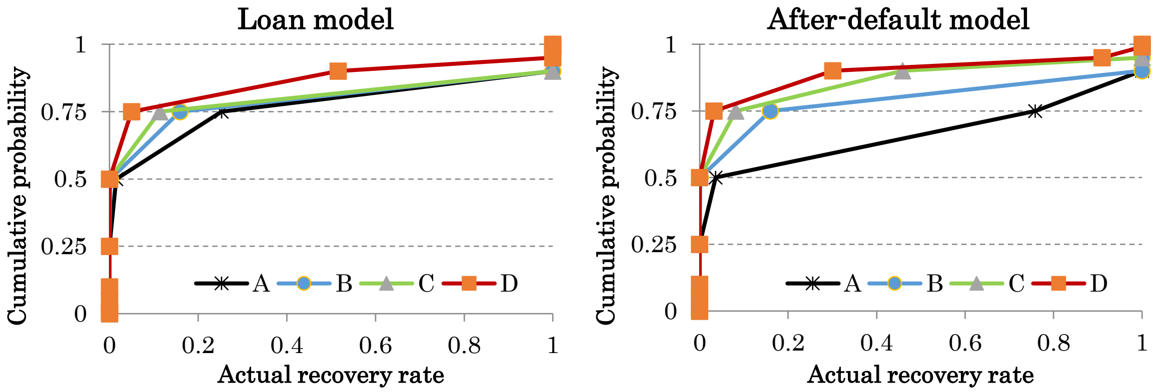

Figure 7: Cumulative distribution of actual RR for loans to industrial corporations

We show the mean actual RR for loans to industrial corporations defaulted in FY 2012 in Table 14.

Table 14 : Mean actual RR for loans to industrial corporations defaulted in FY2012

	Loan model			After-default model		
	N	(ratio)	Actual RR	N	(ratio)	Actual RR
A	1,759	(24%)	22%	2,423	(29%)	31%
B	1,716	(24%)	19%	2,381	(29%)	19%
C	1,919	(27%)	17%	1,895	(23%)	12%
D	1,839	(25%)	12%	1,620	(19%)	9%
All	7,233	(100%)	17%	8,319	(100%)	19%
Shirley-Williams' test	p-value < 0.01			p-value < 0.01		
Kruskal-Wallis test	p-value < 0.01			p-value < 0.01		
Mann-Whitney U test for A&B, B&C, and C&D	p-value < 0.01			p-value < 0.01		

We find the mean actual RRs of each rating are also in a proper order for the loan and after-default models, respectively. The differences between the mean actual RR of ratings 'A' and 'D' are 10% points in the loan model and 22% point in the after-default model. The difference in the after-default model is smaller than those of the in-sample cases in Table 9, due to the decrease in the actual RR of 'A'-rating. However, the difference is relatively large, and it is sufficiently acceptable in practice.

[11] We do not have such a problem in Section 5 because the in-sample data can be divided into equal numbers.

Furthermore, the results of three kinds of statistical tests show that the p-values are less than 1%, and the ranks of ratings are statistically significant at 1% level.

We show the results of actual RR for loans to industrial corporations provided from FY2007 to FY2010 in Table 15, as well as Table 13.

Table 15 : Mean actual RR for loans defaulted in FY2012, provided from FY2007 to FY2010

<table>
<tr><th colspan="2"></th><th colspan="4">Loan model</th><th colspan="4">After-default model</th></tr>
<tr><th colspan="2">FY</th><th>2007</th><th>2008</th><th>2009</th><th>2010</th><th>2007</th><th>2008</th><th>2009</th><th>2010</th></tr>
<tr><td colspan="2">A</td><td>27%
(195)</td><td>21%
(345)</td><td>19%
(433)</td><td>20%
(342)</td><td>30%
(486)</td><td>28%
(592)</td><td>26%
(531)</td><td>25%
(205)</td></tr>
<tr><td colspan="2">B</td><td>24%
(200)</td><td>19%
(333)</td><td>18%
(401)</td><td>17%
(352)</td><td>13%
(265)</td><td>18%
(519)</td><td>18%
(669)</td><td>20%
(585)</td></tr>
<tr><td colspan="2">C</td><td>20%
(185)</td><td>20%
(327)</td><td>15%
(509)</td><td>15%
(430)</td><td>17%
(78)</td><td>9%
(259)</td><td>9%
(486)</td><td>13%
(540)</td></tr>
<tr><td colspan="2">D</td><td>16%
(206)</td><td>15%
(367)</td><td>13%
(449)</td><td>11%
(330)</td><td>5%
(53)</td><td>7%
(156)</td><td>10%
(289)</td><td>11%
(306)</td></tr>
<tr><td colspan="2">All</td><td>22%
(786)</td><td>19%
(1,372)</td><td>16%
(1,792)</td><td>16%
(1,454)</td><td>22%
(882)</td><td>19%
(1,526)</td><td>17%
(1,975)</td><td>17%
(1,636)</td></tr>
<tr><td rowspan="3">S-W test:
p-value</td><td>Level 1</td><td>0.08</td><td>0.02</td><td>0.16</td><td>0.05</td><td rowspan="3">< 0.01</td><td rowspan="3">< 0.01</td><td rowspan="3">< 0.01</td><td>0.06</td></tr>
<tr><td>Level 2</td><td>0.01</td><td>0.04</td><td rowspan="2">< 0.01</td><td rowspan="2">< 0.01</td><td rowspan="2">< 0.01</td></tr>
<tr><td>Level 3</td><td>< 0.01</td><td>< 0.01</td></tr>
<tr><td colspan="2">K-W test: p-value</td><td>< 0.01</td><td>< 0.01</td><td>< 0.01</td><td>< 0.01</td><td>< 0.01</td><td>< 0.01</td><td>< 0.01</td><td>< 0.01</td></tr>
<tr><td rowspan="3">M-W U
test:
p-value</td><td>A&B</td><td>0.15</td><td>0.05</td><td>0.32</td><td>0.10</td><td>< 0.01</td><td>< 0.01</td><td>< 0.01</td><td>0.12</td></tr>
<tr><td>B&C</td><td>0.38</td><td>0.78</td><td>0.01</td><td>0.30</td><td>0.28</td><td>< 0.01</td><td>< 0.01</td><td>< 0.01</td></tr>
<tr><td>C&D</td><td>0.23</td><td>0.02</td><td>0.32</td><td>< 0.01</td><td>0.03</td><td>0.16</td><td>0.37</td><td>0.59</td></tr>
</table>

* The number of defaulted loans is in the parenthesis.

We show the results of Shirley-Williams' test for each level. We find the results of Level 2 and Level 3 are statistically significant at 1% level except one case, whereas the p-values of Level 1 are larger than other levels. The results show that the ranks are statistically significant at least at 10% level except FY2009 in loan model which p-value is 0.16. The Kruskal-Wallis test shows that the p-values are less than 1%, but the orders of mean actual RRs of each rating are unstable, and therefore we need to pay attention to the results concerning whether the ranks are effective. There are twelve combinations which are not statistically significant at 10% level in the Mann-Whitney U test, whereas there are seven combinations which are significant at 1% level. This is also likely to be caused by data deficiency for performing the Mann-Whitney U test as well as Table 13.

7. Conclusion

We develop two models for each type of firms. We analyze the actual RR using the data of 66,928 loans in default unsecured and unguaranteed by third-party to small-sized firms. We construct the RR estimation models using these effective variables by the ordered logistic regression analysis. We find the significant factors affecting the RR through the analysis, and those with large regression coefficients are different in two models for each type of firms. These are (1) guarantee by business owner's family in two models for each type of firms, (2) firm age in two models for industrial corporations, (3) exposure rate at default in the after-default model for each type of firms, (4) obligor's real-estate value minus debt amount, initial loan amount, and white tax return in the loan model for sole proprietorships.

The values of Somers' D for the after-default model is larger than those for the loan model because the EAD rate which has large estimates can be available at time of default. The value of Somers' D for sole proprietorships is larger than that for industrial corporations.

Four ratings are given to the loans based on the score estimated from the RR model, and we examine the performance using the actual RR by rating. We find the appropriate result that the higher the score is, the higher the actual RR is, for two models used for loans to both sole proprietorships and industrial corporations. Furthermore, we calculate the actual RR by rating, and by fiscal year of default and providing loan in order to evaluate the robustness in time series. We find that the mean actual RR of each rating is in a proper order every fiscal year of default and providing loan.

The loan model to the industrial corporations has a smaller difference in the RR between ratings than that to the sole proprietorships. The reason is that non-current assets and liabilities of business owners cannot be taken into consideration in the model for loans to industrial corporations. The Financial Services Agency (2015a) in Japan provides the instruction and advice that non-current assets such as real estate are one of the key factors to evaluate the repayment ability of business owner in classifying the obligors of small firms in self-assessment. This also applies to the estimation of RR after default.

We conduct the Shirley-Williams' multiple comparison test in order to investigate whether the ranks of ratings are statistically significant. The results for in-sample test show the ranks of ratings are statistically significant at 1% level in every fiscal year of default and providing loans in two models for each type of firms. We also perform the Kruskal-Wallis test for comparing the difference between ratings, and the Mann-Whitney U test for the difference between neighboring two groups. The Kruskal-Wallis test also shows there are statistically significant differences at 1% level, and the Mann-Whitney U test shows the significant differences at 1% level except for some cases. We find the model can be effective for practical use.

In addition, we conduct the out-of-sample test for loans defaulted in FY2012. We also derive the appropriate result as well as the in-sample test, and the mean actual RR of each rating is in a proper order. Moreover, we calculate the actual RR in each provided year from FY2007 to FY2010 in order to evaluate the robustness in time series. The mean actual RR of each rating is in a proper order except some cases. We also conduct three statistical tests for out-of-sample data. The results of the Shirley-Williams' test show the p-values are less than 1% for loans defaulted in FY2012 in two models for each type of firms. The ranks of ratings are statistically significant at 1% level in many cases for loans defaulted in FY2012, provided from FY2007 to FY2010 respectively. The Kruskal-Wallis test also shows there are statistically significant differences at 1% level, but there are a lot of cases where the p-values are more than 10%. This is likely to be caused by data deficiency for performing the Mann-Whitney U test, and this is our future research.

Finally, we need to notice an important point when we evaluate the results of out-of-samples based on the statistical tests. We can equalize the number of samples in each rank for the in-sample test. However, each of the number of samples might be different from others in the out-of-sample test, because the actual RRs are dependent on the period from the default event. This may cause the unstable results of the statistical tests. We need to store default and recovery data over long-term period in order to solve this problem.

Our future task is further improvement on the accuracy of the RR estimation. In our paper, we estimate the RR simply by the widely used ordered logistic regression because it is easy to interpret the effect of variables. From now on, we will explore the possibility of using various methods of machine learning such as support vector machines and neural networks, and attempt to improve the accuracy. We hope that this study will be helpful for financial institutions that provide loans to small-sized firms.

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Appendix A Variable list used for Step 3

Financial accounting variables and attribute variables are listed below as candidates for Step 3.

Table A.1: Financial accounting variable list

	Financial accounting variables	S.P.(*1)	I.C.(*2)	abbreviated description
F01	Sales amount	x	x	Sales
F02	Net sales amount	x	x	Net sales
F03	Sales cost	x	x	Sales cost
F04	Gross profit	x	x	Gross profit
F05	Selling, general and administrative expenses	x	x	Selling expenses
F06	Labor cost	x	x	Labor cost
F07	Depreciation cost	x	x	Depreciation cost
F08	Operating profit	x	x	Operating profit
F09	Non-operating expenses	x	x	Non-operating expenses
F10	Interest expenses	x	x	Interest expenses
F11	Profit before income taxes	x	x	Profit before income taxes
F12	Current assets	x	x	Current assets
F13	Cash and deposits	x	x	Cash and deposits
F14	Accounts receivable-trade	x	x	Accounts receivable
F15	Inventories	x	x	Inventories
F16	Other current assets	x	x	Other current assets
F17	Non-current assets	x	x	Non-current assets
F18	Assets	x	x	Assets
F19	Current liabilities	x	x	Current liabilities
F20	Accounts payable-trade	x	x	Accounts payable
F21	Other current liabilities	x	x	Other current liabilities
F22	Non-current liabilities	x	x	Non-current liabilities
F23	Long-term debt	x	x	Long-term debt
F24	Liabilities	x	x	Liabilities
F25	Average monthly principal repayment for long-term debt	x	x	Monthly repayment
F26	Labor costs for representatives and family members	x	x	Labor costs for M
F27	Real-estate value minus debt amount (log value)	x		Real-estate minus debt
F28	Non-current assets minus non-current liabilities (log value)		x	Non-current A minus L

*1 S.P.: Sole proprietorships, *2 I.C.: Industrial corporations

Table A.2: Attribute variable list

	Model	Loan		After-default		
	Attribute variables	S.P.(*1)	I.C.(*2)	S.P.(*1)	I.C.(*2)	abbreviated description
A01	Firm age (*3)	x	x	x	x	Firm age
A02	Number of employers	x	x	x	x	Number of employers
A03	Initial loan amount (log value)	x	x	x	x	Initial loan amount
A04	Guarantee by business owner's family dummy	x	x	x	x	Guarantee (D)
A05	Manufacturing industry dummy	x	x	x	x	Manufacturing (D)
A06	Construction industry dummy	x	x	x	x	Construction (D)
A07	Wholesale and retail trade industry dummy	x	x	x	x	Wholesale (D)
A08	Accommodations, eating and drinking services industry dummy	x	x	x	x	AED services (D)
A09	Medical, healthcare and welfare industry dummy	x	x	x	x	Medical (D)
A10	Service industry dummy	x	x	x	x	Service (D)
A11	Real estate industry dummy	x	x	x	x	Real estate (D)
A12	Transport industry dummy	x	x	x	x	Transport (D)
A13	Loan of working capital dummy	x	x	x	x	Working capital (D)
A14	Repayment period	x	x	x	x	Repayment period
A15	White tax return dummy	x	x	x	x	White tax (D)
A16	Owner's age(*3)	x	x	x	x	Owner's age
A17	EAD rate (EAD divided by initial loan amount)			x	x	EAD rate
A18	EAD (log value)			x	x	EAD

*1 S.P.: Sole proprietorships, *2 I.C.: Industrial corporations, *3 Firm age and owner's age at providing loan are used in the loan model, whereas those at default are used in the after-default model.

Appendix B. EAD-weighted actual RR

B.1 In-sample tests

Table B.1 shows the EAD-weighted actual RRs for loans to sole proprietorships. The EAD-weighted actual RRs of each rating are also given in a proper order in both loan model and after-default model as well as Table 6.

Table B.1: EAD-weighted actual RR for loans to sole proprietorships

Loan model				After-default model			
Rating	N	EAD composition ratio	EAD-weighted actual RR	Rating	N	EAD composition ratio	EAD-weighted actual RR
A	3,695	16%	39%	A	7,441	10%	56%
B	3,694	26%	25%	B	7,445	22%	31%
C	3,694	20%	17%	C	7,450	30%	23%
D	3,695	38%	12%	D	7,436	38%	14%
All	14,778	100%	21%	All	29,772	100%	25%

The EAD-weighted actual RRs for loans to industrial corporations are shown in Table B.2, and they are also in a proper order as well as sole proprietorships in Table B.1.

Table B.2: EAD-weighted actual RR

Loan model				After-default model			
Rating	N	EAD composition ratio	EAD-weighted actual RR	Rating	N	EAD composition ratio	EAD-weighted actual RR
A	7,465	26%	15%	A	9,290	13%	25%
B	7,466	23%	13%	B	9,288	27%	14%
C	7,464	25%	10%	C	9,292	32%	9%
D	7,465	25%	7%	D	9,286	28%	7%
All	29,860	100%	11%	All	37,156	100%	12%

B.2 Out-of-sample tests

We show the EAD-weighted actual RRs for loans to sole proprietorships in the out-of-sample tests. Table B.3 shows they are in a proper order as well as Table 12.

Table B.3 : EAD-weighted actual RR for loans to sole proprietorships defaulted in FY2012

	Loan model			After-default model		
	N	(ratio)	Actual RR	N	(ratio)	Actual RR
A	1,017	(22%)	41%	1,258	(20%)	52%
B	1,041	(23%)	25%	1,729	(27%)	31%
C	1,149	(25%)	17%	1,841	(29%)	21%
D	1,357	(30%)	12%	1,477	(23%)	13%
All	4,564	(100%)	20%	6,305	(100%)	22%

Table B.4 shows the EAD-weighted actual RRs for loans to industrial corporations defaulted in FY2012 are also in a proper order as well as sole proprietorships in Table B.3.

Table B.4 : EAD-weighted actual RR for loans to industrial corporations defaulted in FY2012

	Loan model			After-default model		
	N	(ratio)	Actual RR	N	(ratio)	Actual RR
A	1,759	(24%)	17%	2,423	(29%)	20%
B	1,716	(24%)	13%	2,381	(29%)	14%
C	1,919	(27%)	11%	1,895	(23%)	11%
D	1,839	(25%)	7%	1,620	(19%)	8%
All	7,233	(100%)	12%	8,319	(100%)	13%