Time-series analysis of truncated realized volatility

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***Abstract*—Accurate measures and good forecasts of volatility is a very critical issue for risk management, pricing derivatives, and asset allocation problems. Andersen and Bollerslev (1998) proposed realized volatility (RV), which is estimated as the sum of squared values of the intraday log-returns, and it is model-independent volatility. In recent years, the RV has been actively studied because high-frequency data is easily available for researchers. However, it is difficult to measure an accurate volatility since the extremely large fluctuations such as jumps are observed in the market. Then Ait-sahalia and Jacod (2012) proposed truncated realized volatility (TRV) to overcome the problem. The TRV is a RV calculated employing absolute values of return below a certain threshold, and they conduct theoretical analysis on how to determine the threshold. In this paper, we examine the methods of selecting the optimum threshold of TRV using TOPIX (Tokyo Stock Price Index), and propose various measures of selecting thresholds. We also evaluate the volatility measures through time-series analysis based on the time-series stationarity and predictability because it is important to improve the prediction accuracy using time-series models in managing risk in financial institutions. We have good results for the model which changes the threshold in response to the market trend. Furthermore, we conduct simulations using the Merton jump diffusion model because it is important to accurately estimate the parameters in the price process model when pricing derivatives such as options. We find that the proposed method of determining threshold gives the good expression of the actual price process.**

***Index Terms*—financial engineering, realized volatility, time-series analysis**

I. Introduction

Volatility llerslev [1] proposed realized volatility (RV), which is calculated as the sum of squared values of the intraday log-returns, and it is model-independent volatility. As the number of observed returns increases, the RV converges to the time integral value of the volatility in the diffusion process. In recent years, the RV has been actively studied because high-frequency data is easily available for researchers. However, it is important to estimate volatility appropriately considering the effect of jumps because many studies point out the existence of jump in actual asset price fluctuations.

Mancini [3] proposed to estimate the RV using the data that excludes jumps in order to estimate the volatility of the Brownian motion components in the price process accurately. Specifically, he proposed the RV calculated employing absolute values of return below a certain threshold, which is called truncated realized volatility (TRV). Ait-sahalia and Jacod [4] conducted theoretical analysis on how to determine the threshold. Furthermore, Yoshida [10] measured the TRV of TOPIX (Tokyo Stock Price Index) using the method of determining the threshold of Ait-sahalia and Jacod [5], and considered the relationship between the observation time interval and TRV. However, it points out that there is a problem in selecting the optimal threshold.

In this paper, we focus on the selection of the optimal threshold in TRV, and propose volatility measures based on various thresholds. Moreover, we define an accuracy is evaluated by the time-series stationarity and predictability to estimate of volatility, and attempt to estimate the volatility of Brownian motion accurately through the time series analysis.

Furthermore, we employ the volatility measure with high predictive capability in each phase based on technical indicators, and show the importance of selecting the appropriate threshold according to the market trend. We also conduct simulations using the Merton jump diffusion model in order to verify that the proposed method of determining threshold gives the good expression of the actual price process.

II. Realized Volatility

A. Derivation of realized volatility

Intraday return at time j in day i is described as $r_{j,i} = p^*\left(i-1+\frac{j}{M}\right) - p^*\left(i-1+\frac{j-1}{M}\right), j = 1,2,...,M$, where $p^*(t)$ is log-price at time t and $0 \leq t \leq T$. T is the terminal period, and the realized volatility of day i can be expressed as

$$RV_i = \sum_{j=1}^{M} r_{j,i}^2 \tag{1}$$

where M is the number of observations of intraday returns per day.

Although the RV is very simple calculation, there is theoretical background that the RV is an appropriate estimate of volatility. Suppose that $p^*(t)$ follows the process of

$$dp^*(t) = \mu(t)dt + \sigma(t)dW(t) \tag{2}$$

where $\sigma(t)$ is spot volatility. The volatility over a given period is calculated as the integral of the spot volatility such that

$$\sigma^{2*}(t) \equiv \int_0^t \sigma^2(u)du, \tag{3}$$

and it is called integrated volatility. If p^*(t) is semi-martingale, the RV converges to integrated volatility as the number of observations of intraday return increases. This shows the RV is a consistent estimator of integrated volatility.

B. Issues of realized volatility

While the RV has excellent features that it is calculated by a very simple estimation method and becomes a consistent estimator of integrated volatility, it has the following issues.

Errors due to returns outside trading hours

The RV is affected by the overnight and lunch break returns outside trading hours, and it may increase the time discretization error. In order to adjust the effect, we use the method of multiplying the RV calculated excluding overnight and lunch break returns by the ratio of the mean of RV to the variance of the daily log-return(RVHL hereafter).

Autocorrelation of price fluctuation

Shibata [7] insisted that the rate of return of stock index such as Nikkei 225 is autocorrelated because it is calculated as the average of individual stock price separately affected by the information which flows into the market. If the autocorrelation decays quickly, we can ignore the effect of the autocorrelation and the RV is calculated by extending the interval of return observation. In this paper, we choose the five-minite interval for intraday returns in the calculation of RV and TRV as well as many previous studies.

Jumps in price fluctuations

Many studies have shown the existence of jumps in the actual price process. The RV is not a consistent estimator of integrated volatility for the price process with jumps, even if the number of observations M converges to infinity. One of the solutions for the price process with jumps is to use the truncated realized volatility (TRV) proposed by Ait-sahalia and Jacod [4].

C. Derivation of truncated realized volatility

The truncated realized volatility (TRV) of day i is calculated by

$$TRV_i = \sum_{j=1}^{M} r_{j,i}^2 1_{\{|r_{j,i}| \le u_i\}}, \tag{4}$$

where u_i is a positive threshold of day i. $1_{\{a\}}$ is an indicator function that is equal to 1 if the condition a is satisfied and 0 otherwise. The function attempts to give the accurate integrated volatility of the Brownian motion by removing the jump returns and adding up $r_{j,i}$ whose absolute value is below the threshold. In the determination of the optimal threshold, it is necessary to consider the trade-off relationship between removing the effect of the jump components and maintaining the fluctuations expressed by the Brownian motion. In the next section, we explain the method of determining the threshold by Ait-Sahalia and Jacod [4] and the proposed method in this paper.

D. Method of determining the threshold

Ait-Sahalia and Jacod [4] determines the threshold by the following method. We use some of the same descriptions as in Yoshida [10]. First, suppose that $r_{j,i}$ follows a diffusion process without jumps, and when calculating TRV with the threshold u_n, the difference between the estimates of RV and TRV, A_T, is expressed as

$$A_T = \int_0^T c(s) g\left(\frac{u_n}{\sqrt{c(s)\Delta_n}}\right) ds \tag{5}$$

where $c(s) = \sigma^2(s)$, $g(u) = \int_{|x|>u} x^2 \phi(x) dx$ and $\phi(x)$ is the probability density function of the standard normal distribution. Δ_n indicates the observation time interval. Also, the standard deviation of RV and TRV estimate is $\sqrt{2\Delta_n \int_0^T c^2(s)ds}$. If A_T is limited below the θ times of this standard deviation, Equation (6) is obtained.

$$\int_0^T c(s) g\left(\frac{u_n}{\sqrt{c(s)\Delta_n}}\right) ds \le \theta \sqrt{2\Delta_n \int_0^T c^2(s) ds} \tag{6}$$

We set $\theta = 0.1$ as well as Ait-Sahalia and Jacod [4], and other parameters as follows.

$$c_{max} = sup(c(s) : s \in [0, T]) \tag{7}$$

$$c_{min} = inf(c(s) : s \in [0, T]) \tag{8}$$

$$c_{aver} = \frac{1}{T} \int_0^T c(s) ds \tag{9}$$

$$\zeta \ge \frac{c_{max}}{c_{min}} \tag{10}$$

Since $g(\cdot)$ is a decreasing function, Equation (6) can be expressed as

$$g\left(u_n \sqrt{\frac{\zeta}{c_{aver}\Delta_n}}\right) \le \frac{\theta}{\zeta}\sqrt{\frac{2\Delta_n}{T}} \tag{11}$$

We set $\zeta = 3$ as well as Yoshida [10]. Also, let z_η be the value where the two-sided probability of the standard normal distribution is η. Let u_η be the threshold of excluding the η percentage of intraday returns in calculating the TRV, and we estimate c_{aver} as

$$\hat{c}_{aver} = \frac{1}{T(1 - g(z_\eta))} \hat{C}(\Delta_n, u_\eta)_T \tag{12}$$

We set $\eta = 0.25$ as well as Yoshida [10]. $\hat{C}(\Delta_n, u_\eta)_T$ is the sum of TRV in T days under the condition that observation time interval is Δ_n and the threshold is u_η. The parameter values except u_n in Equation(11) are determined. Therefore we can calculate the maximum of u_n that satisfies Equation(11), and the TRV is calculated using Equation(4). The TRV derived from the method by Ait-Sahalia and Jacod [4] is called "TRV-ait" in this paper. Next, we propose two methods of determining the threshold. The first method assumes that intraday returns are normally distributed. The TRV derived from the method is called "TRV-norm". The specific process is shown below.

1) Determine the observation days T and the observation time interval Δ_n and standardize the log-return data of the intraday price for T days so that the average is zero and the standard deviation is one.

2) Exclude the data of the standardized log-return in descending order of the absolute value and conduct the KS (Kolmogorov–Smirnov) test on the data sequentially. The distributions compared in the KS test are normal distributions whose mean and standard deviation are equal to those of the standardized log-return data.
3) Calculate the p-value in each KS test, count the number of data excluded in the test with the largest p-value, and denote it by p_{max}.
4) In the data of squared log-return of the intraday price for T days, the threshold value is the square root of the p_{max}-th data in descending order.

The second method is to exclude a certain percentage of return data. because we assume that the price fluctuation contains a certain percentage of jumps. TRV obtained by the method is called "TRV-fix". The specific process is shown below.

1) Determine the observation days T and the observation time interval Δ_n and obtain the number of log-return data N of the intraday price for T days and the fixed rate P.
2) Let p_{fix} be the value of $N \times P$ rounded to an integer.
3) In the data of squared log-return of the intraday price, the threshold value is the square root of the p_{fix}-th data in descending order.

TRV-fix($x\%$) is TRV excluding $x\%$ of the intraday log-return data.

III. Evaluation of volatility measures in time-series analysis

A. Pre-analysis

We compare the predictive capabilities of RV, RVHL, TRV-ait, TRV-norm, and TRV-fix through time series analysis. As a pre-analysis, we determine which time series model is used in the analysis. First, we calculate each TRV-fix that excludes data with 10% increments in between from 0 to 90%, and conduct the unit root test (Phillips Perron test). The result shows the p-value in the unit root test is less than 0.01 for all volatility measures, and the null hypothesis that the time series data follows a non-stationary process is rejected at a 1% significance level. Therefore, we employ the time series model with a stationary process for the analysis, and select the ARMA (2,1) model according to AIC(Akaike's Information Criterion) in this paper. The ARMA(p, q) (Autoregressive Moving Average) model in the data series y_i is expressed as

$$y_i = \epsilon_i + \sum_{j=1}^{p} \alpha_j y_{i-j} + \sum_{j=1}^{q} \beta_j \epsilon_{i-j} \tag{13}$$

where ϵ_i is the error term, p is the number of autoregressive terms, and q is the number of moving-average terms.

B. Evaluation of volatility measures based on loss function

We conduct the time-series analysis using the ARMA model determined in the pre-analysis, and compare the predictive capability of all volatility measures in the following process. First, each volatility measure is calculated using one year intraday return data from the first day to the 250th day in the data period. Then, we estimate the parameters of the time series model using the volatility measure, and predict the volatility $\hat{\sigma}_i$ on the 251st day. The threshold value of TRV and the parameter of RVHL observed on the 251st day are determined by the data from the first day to the 250th day. Next, we calculate the predicted values in the same way using the rolling windows from the second day to the 251st day, the third day to the 252nd day, and so on, and evaluate the predictive capabilities of volatility measures based on the loss function. In order to verify the robustness of predictive capability of each measure, the data period is divided into two periods; Period 1 from January 5, 2006 to December 30, 2013 and Period 2 from January 6, 2014 to July 31, 2018. The same rolling predictions are performed in both periods. We select the loss functions of RMSPE(Root Mean Squared Percentage Error) and MAPE(Mean Absolute Percentage Error) which calculate the rates of relative error between the predicted value $\hat{\sigma}_i$ and the observed value σ_i because the scale of TRV is greatly affected by the threshold and the loss function based on the absolute error is not appropriate for evaluation. The calculation formula of RMSPE and MAPE are shown as

$$RMSPE = \sqrt{\frac{1}{T} \sum_{i=1}^{T} \left(\frac{\sigma_i - \hat{\sigma}_i}{\sigma_i} \right)^2} \tag{14}$$

$$MAPE = \frac{1}{T} \sum_{i=1}^{T} \left| \frac{\sigma_i - \hat{\sigma}_i}{\sigma_i} \right| \tag{15}$$

The data used in the analysis is shown in Table I. The results are shown in Table II and Fig. 1. Fig. 1 includes the constant values of RMSPE and MAPE of TRV-norm for comparison to those of TRV-fix.

TABLE I
DATA USED IN TIME-SERIES ANALYSIS IN PERIOD 1 AND PERIOD 2

	Period 1	Period 2
Data period	January 5, 2006 to December 30, 2013	January 6, 2014 to July 31, 2018
Sample number	1959	1121
Volatility measure	RV, RVHL, TRV-ait, TRV-norm, TRV-fix	
Time-series model	ARMA(2,1)	
Method of estimation	MLE	

TABLE II
RESULT OF LOSS FUNCTION VALUES

	Period 1		Period 2	
	RMSPE	MAPE	RMSPE	MAPE
RV	6.8361	2.6609	2.0905	1.3117
RVHL	0.8316	0.5386	1.5388	1.0041
TRV-ait	0.4519	0.3120	0.5362	0.3791
TRV-norm	0.2914	0.2075	0.3314	0.2274

RVHL has a higher predictive capability than RV in both periods and both loss functions because of the effect of adjusting the returns of overnight and lunch break in RVHL. This result is consistent with the previous research by Watanabe

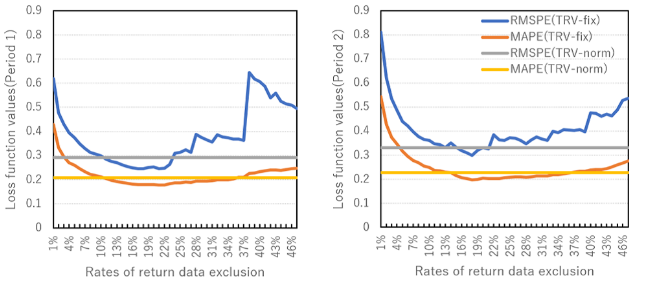

Fig. 1. Comparison of loss function values of TRV

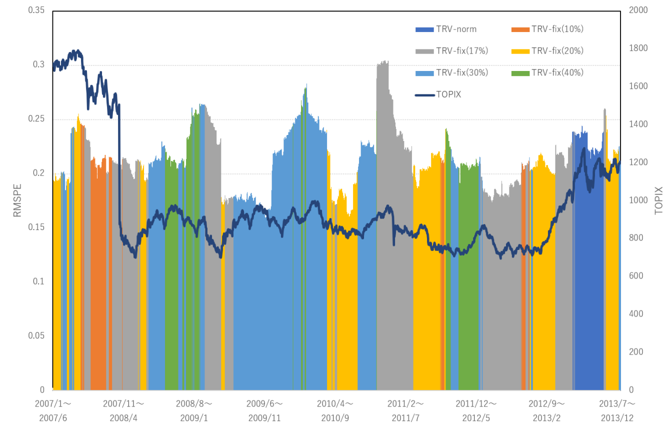

Fig. 2. Changes in RMSPE and TOPIX

[11] that the predictive capability of RVHL exceeds that of RV. TRV-norm has a higher predictive capability than TRV-ait in both periods and both loss functions. This result shows that TRV-ait is not able to remove the jump appropriately. Fig. 1 shows that the predictive capability of TRV-norm is worse than the one with the smallest loss function in TRV-fix. TRV-fix has the higher predictive capability than TRV-norm when the percentage of exclusion is about 10% to 20% in RMSPE and 10% to 35% in MAPE. Fig. 2 shows the RMSPE value for the volatility measure with the excluded percentage that minimizes RMSPE every 100 days in Period 1, and the change in TOPIX. We choose the TOPIX price at the middle of the period in the calculation of RMSPE for simplicity. We find that the measures that minimize the loss function value are different over time. Furthermore, the excluded percentage of calculating TRV-fix is relatively small during the periods of large price fluctuations such as periods of a large downward trend from November 2007 to April 2008, and an upward trend from September 2012 to February 2013. Conversely, the excluded percentage tends to increase for the period of small price fluctuations from June to November in 2009. This shows that there exists the relationship between the market trend and the excluded percentage of intraday return data. Therefore, it is expected that the prediction capability of volatility is improved by dividing the market situations into multiple phases and selecting the appropriate volatility measures in each phase. We also find that the volatility measures of minimizing the loss function values are different over time in Period 2.

IV. Selection of Volatility Measures According to Market Trend

A. Determination of measures with high predictive capability in each phase

We divide the market situations into multiple phases based on technical indicators and conduct time-series analysis on each phase because it is expected that the predictive capability in time series analysis can be improved by selecting appropriate volatility measures according to the market trend. We evaluate volatility measures based on loss functions. First, we explain the technical indicators used in this paper. We select VHF(Vertical Horizontal Filter), psychological line, and Donchian channel as technical indicators because it is expected that there exists the relationship between the market trend and the excluded percentage of rate of return data as shown in the previous section.

(1) VHF

The VHF is an indicator that distinguishes whether a market is a trend market whose price movement is large or a range market whose price movement is small. The market is judged as trend market if VHF value is more than 0.3 and range market if the VHF value is less than 0.3. The VHF is calculated by dividing the difference between highest and lowest prices for the last n days by the sum of absolute value of difference from previous day for n days. The value is multiplied by 100. We choose $n = 28$ used in general.

(2) Psychological line

The psychological line is an indicator that attempts to express the investor's psychology. The market situation is judged as overbought when the percentage of days of the price increase in the last n days exceeds 75%, oversold when it falls below 25%. We divide the market situations into three phases; overbought, oversold, standard(neither overbought nor oversold). We set $n = 12$ used in general.

(3) Donchian channel

The Donchian channel is an indicator that distinguishes whether the market is an uptrend or a downtrend. The market situation is judged as an uptrend when the market price is higher than the Donchian channel calculated by prices in the last n days, downtrend when the price is lower than the Donchian channel. The Donchian channel is calculated as a half of the sum of highest and lowest prices for the last n days. We set $n = 20$ used in general.

Next, we conduct the time series analysis in each phase, and calculate the loss function after dividing the market situations into multiple phases by technical indicators in Period 1. We use the same method of time series analysis as Section III, and select the volatility measure with the smallest loss function in each phase.

The results are shown in Table III. In the case of the VHF and psychological line, the relatively large excluded percentages lead to the high predictive capabilities in the market without trend. This is because the TRV is sensitive to a few large fluctuations in the market without trend and range market with small price fluctuation, and the excluded percentage is relatively large in order to reduce the loss function value. In the case of the Donchian channel, the large excluded percentage in the uptrend market and low percentage in the downtrend market lead to the high predictive capability. It is considered that the tail returns do not need to be excluded because the price fluctuation is relatively large and the fluctuation of TRV becomes small in the downtrend market. Actually we find that the the retrun distribution of downtrend has fatter tails compared with uptrend.

TABLE III
VOLATILITY MEASURES WITH HIGH PREDICTIVE CAPABILITY IN EACH PHASE

	VHF		Donchian channel	
	trend market	range market	up-trend	down-trend
RMSPE	TRV-fix (17%)	TRV-fix (22%)	TRV-fix (22%)	TRV-fix (17%)
MAPE	TRV-fix (18%)	TRV-fix (33%)	TRV-fix (22%)	TRV-fix (17%)
	Psychological line			
	over-bought	standard	over-sold	
RMSPE	TRV-fix (12%)	TRV-fix (21%)	TRVfix (17%)	
MAPE	TRV -norm	TRV-fix (21%)	TRV-fix (17%)	

B. Evaluation by loss function when using different measures for each phase

The selection of volatility measures affects the results. We examine what volatility measure has a high predictive capability for each phase which is classified by technical indicator. We compare the method of switching different volatility measures according to the phase with that of using a single measure in Period 2. The result is shown in Fig. 3. We find the predictive capability gets higher by using technical indicators than a single measure. In particular, the method of switching the volatility measure based on the phase classified using the Donchian channel gives a good predictive capability because the market trend is divided into two phases of uptrend and downtrend equally. On the other hand, switching by the VHF and psychological line does not give a good predictive capability because the percentage of range market assigned by the VHF is about 20% and the percentage of overbought and oversold assigned by the psychological line are respectively less than 10%, which are very small. It is considered that the predictive capability can be improved by changing the boundaries of the technical indicators, such as 0.3 in VHF and dividing the market situations into multiple phases with unbiased proportion.

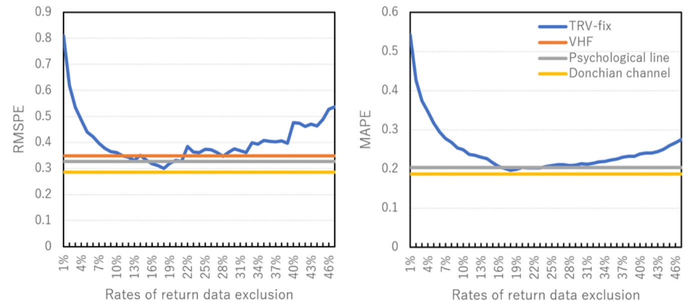

Fig. 3. Results of switching different volatility measures according to the phase and using TRV-fix constantly

V. SIMULATION USING MERTON JUMP DIFFUSION MODEL

In pricing derivatives such as options, it is important to estimate parameters in the price process accurately. Therefore, we examine whether the price process including the volatility measures based on the method proposed in the previous section actually work well by conducting the simulation using the Merton jump diffusion (MJD) model [9].

A. Analysis method

The MJD model is expressed by

$$\frac{dS_t}{S_t} = (\mu - k\lambda)dt + \sigma_B dZ_t + (J-1)dN_t \tag{16}$$

where N_t is the Poisson process, $J-1$ is the relative jump size[1], μ is the expected return of the underlying asset, $k = E[J-1]$, λ is the intensity of the Poisson process, and σ_B is the volatility of the Brownian motion. Instead of continuous time setting, we rewrite the equation in the following discrete time setting for simulation.

$$\ln\left(\frac{S_{t+1}}{S_t}\right) = \mu_B + \sigma_B Z_t + \sum_{n=1}^{N_{t+1}-N_t} J_n \tag{17}$$

$$\ln J_n \sim \Phi(\mu_J, \sigma_J^2) \tag{18}$$

where $\mu_B \equiv \mu - \lambda k - \sigma_B^2/2$, μ_J is the expected jump value, and σ_J is the jump volatility.

Craine, *et al.* [2] use the likelihood function of the MJD model to estimate parameters that maximize the likelihood. We often fail to obtain the parameters regardless of using the optimization tool of MATLAB, because those are heavily dependent on the initial value. Therefore we determine to employ a simple method to estimate parameters, and conduct the simulation in the same two periods as those in the previous section. We use TRV-ait, TRV-norm, TRV-fix(17%), TRV-fix(21%), and TRV of switching different volatility measures according to the market phase based on technical indicator introduced in Section IV. The analytical process is as follows.

1) The data in the period of 250 days prior to the first day in Period 2 is used to determine the excluded percentage of return data for each volatility measure. Regarding the

[1]When the underlying asset price goes from 100 to 80, $J = 0.8$.

TRV of switching different volatility measures according to the phases classified by technical indicators, we use the measure with the smallest RMSPE in each phase during Period 1.
2) Let μ_j and σ_j be the average and standard deviation of the log-return of the data which jumps are excluded. We calculate the cumulative return of the five-minute interval of the data excluding jumps in the calculation of μ_j every day. We denote the average value of 250 days by μ, the average of TRV by σ_B, and the excluded percentage of return data in calculating TRV by λ, and all parameters in MJD model is calculated.
3) Using the MJD model with parameters determined, we generate 10,000 paths of closing prices on the day following the last day of training data, and obtain the VaR. VaR with $x\%$ is expected maximum loss with probability of $x\%$.
4) We calculate the percentage of actual prices below the VaR in the simulation for each volatility measure by repeating the above procedure for all days in Period 2. The closer this value is to the confidence level of VaR, the more appropriately the actual price process is expressed. We use the VaR with 1% and 5% loss tolerance.

VI. SIMMULATION RESULT

Table IV shows the simulation results for the MJD model. In the case of VaR(1%), the closest to 1% loss tolerance is 3.88% of Donchian channel and TRV-fix(21%). The percentages of other technical indicators are larger than that of TRV-norm, and relatively different from 1%. In the case of VaR(5%), 7.37% of VHF is the closest to 5% loss tolerance, and the Donchian channel that is closest to 1% loss tolerance in VaR(1%) is the second closest to 5% loss tolerance. The method of switching the volatility measure by the Donchian channel is able to express the actual price process appropriately.

The percentage below the VaR are beyond 1% or 5% for each volatility measure in both the VaR (1%) or VaR (5%) respectively. When the TRV is incorporated into the parameter of MJD model, it is expected that the VaR becomes an optimistic estimate.

TABLE IV
SIMMULATION RESULT USING MJD MODEL

	VaR(1%)	VaR(5%)
TRV-ait	9.57%	15.52%
TRV-norm	4.14%	8.28%
TRV-fix(17%)	4.40%	7.63%
TRV-fix(21%)	3.88%	7.50%
VHF	4.27%	7.24%
Psychological line	4.27%	7.50%
Donchian Channel	3.88%	7.37%

VII. CONCLUSION

In this paper, we evaluate the volatility measures based on the predictability in time series analysis. The predictive capability increases in the order of RV, RVHL, TRV-ait, and TRV-norm, and in the range of 15 to 20% excluded percentages, the predictive capability of TRV-fix is higher than TRV-norm. In addition, we show that the predictive capability of volatility is improved by using the volatility measure appropriately according to the market trend. There is a tendency to increase the predictive capability by decreasing the excluded percentage of data in the trend market and increasing the percentage in the range market. In the trend market, the predictive capability is improved by increasing the excluded percentage of data relatively during the uptrend and decreasing the percentage during the downtrend. In particular, switching the volatility measure based on the phase determined by the Donchian channel shows good results in both predictive capability and accuracy of price process, and we achieved to estimate the volatility of the Brownian motion acculately.

In the future tasks, we attempt to improve the selection of jump model because the VaR becomes an optimistic estimate for the MJD model selected for simulation. Furthermore, we need to introduce new technical indicators so that the market situations can be more appropriately classified into multiple phases and increase the ability to predict volatility.

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